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CONTENTS

Biology. — *The influence of X-rays on fertilized axolotleggs of different ages.* By E. WINKLER—JUNIUS and Dr. B. J. VAN DER PLAATS. (From the Psychiatric-Neurological Clinic at Utrecht. Director Prof. Dr. L. BOUMAN.)

(Communicated at the meeting of March 23, 1929).

The motive for our investigation lay in the question in how far histological changes occur after X-raying the nervous tissue. Although this question has scarcely been approximated after terminating our experimental work, yet a series of experiments revealed to us some facts, entitling to communication.

By X-raying the caviabrain, either partially or totally, during different times of exposure, we did not obtain the results we expected, on the contrary, we did not find any changes in the nervous tissue, even not after administering very large doses of X-rays e.g. 20 H. E. D., soft type.

The removal of the chromatine in the PURKINJE-cells under the influence of radium, described by Dr. T. MOROWOKA and Sir FRED. W. MOTT could also be demonstrated in the PURKINJE-cells of the animals we X-rayed (photo I.) as well as in the PURKINJE-cells of the control-animals. Therefore we do not agree with the opinion of the two authors, that the removal of the chromatine was due to the radiumemanations.

Examining a series of normal PURKINJE-cells along a circumvolution of the cerebellum of a normal cavia, one often detects a loss of chromatine in the cells at the base of the circumvolution, whilst in the cells lying along the side of the circumvolution the chromatine has accumulated in the shape of a disk on one side of the nucleus (photo 2.), the stand of which disk however varies with respect to the nucleus in following the PURKINJE-cells to the top of the circumvolution.

The PURKINJE-cells of the irradiated animals too showed this phenomenon. Although we applied X-rays instead of radiumrays, and the removal of the chromatine, described by MOROWOKA and MOTT was obvious in the PURKINJE-cells of our normal animals as well as in the PURKINJE-cells of the X-rayed animals, we cannot but draw the conclusion that this phenomenon possibly depends on the fixation of the material or on the direction in which the sections through the circumvolution were made.

As the X-raying of the caviae did not give any more trustworthy results, we decided to use a lower species of animals for the next experiments viz. the developing forms of the axolotl, 1 : because the breeding of these animals in the laboratory scarcely gives any difficulty, 2 : because the

development of these animals is a very slow one and therefore definite stages of development can easily be chosen.

We limited the X-raying in such a way, that:

1. We always took a definite dose, measured as precisely as possible.
2. Circumstances were completely reproduceable.

The X-ray apparatus consisted of an oilinductor (SIEMENS), connected with a COOLIDGE X-ray tube. By previous experiments it had been stated, that the dose during the time of exposure remained constant.

The eggs of the axolotl or the axolotl larvae were put into a small basin of celluloid of the measurement 7 × 9 cm, in which the water was about 1½ cm high. The basin itself rested on a celluloidfilm, stretched over an opening in the table, in such a way, that the X-ray beam, emitted by the RÖNTGEN-bulb through a diaphragm of 9 × 12 cm aperture, diaphragmized at a distance of 25 cm from the focus, could not strike the table. These precautions had been taken to eliminate the secundary rays. Operating in this way, the larvae in the basin were struck, 1. by the primary beam over a field of 9 × 12 cm at a distance of 23 cm from the focus, 2. by the secundary rays emitted from the water in the basin. The distance between the bottom of the basin and the focus was 25 cm.

The tension was measured at the terminals of the RÖNTGEN-bulb with a high-tensionmeter plate-point. The dose was measured with a verified ionometer. Only once a dose was administered to the larvae.

Four methods of irradiation were applied, numbered as follows:

I. Intensity of current 2½ mA. Potential difference 120 KV. Screened by: 1 mm of alluminium, 4 mm of wood.

Time of exposure: 3 min. Dose 90 R (BEHNKEN).

II. Equal to 1. Time of exposure however 10 min. Dose 300 R (BEHNKEN).

III. Intensity of current 2½ mA. Potential difference 180 KV.

Screened by: 4 mm of wood, ½ mm of zinc, 3 mm of alluminium.

Time of exposure 10 min. Dose 95 R (BEHNKEN).

IV Equal to III. Time of exposure 30 min. Dose 285 R (BEHNKEN).

The doses I and III are, ionometrically measured nearly the same, just as the doses II and IV. The difference between method I and III, as well as between method II and IV is only, that with method I and II soft rays are emitted, with method III and IV hard ones.

The eggs of the axolotls were submitted to irradiation at different stages of development. In all points the control objects underwent the same treatment, except that they were not submitted to irradiation.

Having defined our experiments in the way, mentioned above, we could put the following questions:

I. If a well defined dose of X-rays is administered to a fertilized axolotlegg, what is then the influence of the age at which the irradiation takes place?

II. In how far do the results differ when the ionometric doses are different.

Method I and II might solve the latter problem for soft rays, method III and method IV for hard rays.

The ionometric doses in method I and III as well as in method II and IV being the same, only the quality of applied rays being different, it might be possible to detect a difference in effect between soft rays and hard ones.

The fertilized axolotleggs underwent irradiation at the following ages

Series A: half a day after fertilization
7 eggs with method I. signed A I.
5 eggs with method II. signed A II.
5 eggs with method III. signed A III.
6 eggs with method IV. signed A IV.

Series B: 1½ days after fertilization (stage of blastula).
5 eggs with method I. signed B I.
5 eggs with method II. signed B II.
5 eggs with method III. signed B III.
5 eggs with method IV. signed B IV.

Series C: 2½ days after fertilization.
6 eggs with method I. signed C I.
5 eggs with method II. signed C II.
4 eggs with method III. signed C III.
4 eggs with method IV. signed C IV.

Series D: 4½ days after fertilization (stage in which the neural tube is closing and the mesoderm begins to develop).
4 eggs with method I. signed D I.
5 eggs with method II. signed D II.
5 eggs with method III. signed D III.
4 eggs with method IV. signed D IV.

Series E: 6½ days after fertilization (the optic vesicle is formed, as well as the vacuolized notochord).
6 eggs with method I. signed E I.
4 eggs with method II. signed E II.
4 eggs with method III. signed E III.
4 eggs with method IV. signed E IV.

It would lead us too far to describe circumstantially these 98 eggs, although they were all cut in series and examined.

Only the principal deviations, caused by the RÖNTGEN irradiation in different stages of development will be described and will be arranged systematically.

The first constant deviations of development, appearing in the fertilized

axolotlegg, having been X-rayed one time with one of the doses mentioned above and before the 7th day after fertilization are as follows :

I. A retardation in the development of the embryo : that is to say : the organs formed respectively arise later than is normally the case.

II. The different organs are smaller than they are in the same stage in normal development : sections through these organs prove that they consist of a smaller number of cells than the organs of normal embryos in the same stage.

III. A degeneration of the different tissues may be found : that is to say : not the irradiated cells themselves, but the daughtercells generated by them may be found in a degenerated state.

To prove these theses we limit ourselves to the description of two different irradiationproofs.

I. *Proof of irradiation .*

Two eggs were irradiated 1½ days after fertilization with method III (series B III), and were preserved 8 days after the irradiation.

Both the eggs were in the same stage of development, which stage is shown in the photos 3 and 5. The brainvesicles, the optic vesicle, the auricular one, as well as the olfactory organ were present. The optic vesicle however was still in connection with the brainvesicle by an epithelial tube (photo 3). A lens was only formed on one side of the embryo. The first traces of the heart were seen (photo 5).

Compared with a normal embryo of 9 days old, the difference is obvious. In the latter, the optic vesicle is totally separated from the brainvesicle. Besides, the lens lying ventro-laterally in the invaginated optic vesicle, possesses already lensfibres, whilst the retinal wall shows different celllayers (photo 4). In the two irradiated embryos heartcells have scarcely arisen and only a slit is found in the midst of the ventral yolkmass, not yet covered with an endothelial layer of cells (photo 5). The normal larve of 9 days, on the contrary possesses a distinct endothelial heart with curvature (photo 6).

In the irradiated larve the liver has not yet been formed, whilst the epithelium of the mouth has not yet been differentiated, in contrast with the normal larve.

Both the irradiated larvae had reached almost the same stage of development, so they had almost the same retardation in their development, a fact that could always be stated whenever more than one larva was irradiated at the same stage and with the same method.

This first irradiation experiment chiefly proves that the development undergoes a retardation. Degeneration of tissues just formed, could not yet be found after 9 days.

II. *Irradiationproof.*

Three eggs were X-rayed 2½ days after fertilization with the same

method III, (C III), however the larvae were preserved 38 days after irradiation instead of 8 days Compared with a normal axoloid of 40 days, it was found : I. that the irradiated larvae were nearly half as small as the normal ones ; II. that the development of the different tissues had not gone so far as was the case in the normal ones. As photo 8 shows, the normal optic vesicle of an axolotl of 40 days has a retinal wall consisting of different celllayers, whilst the optic vesicle of the irradiated larvae (photo 7) scarcely has a differentiation of the retina. Moreover, the largest section through the optic vesicle of the irradiated larva shows a much smaller size than that of the normal one. The heart of the normal larva is more than two times as big as that of the irradiated one. The distinct development of muscular fibres, as seen in the normal heart, is not to be detected in the heart of the irradiated larva (photo 9, and photo 10).

The dorsal layer of muscular fibres round the notochord and the medulla spinalis is of much larger size in the normal axolotl of 40 days than is the case in the irradiated one. Also in the normal one is the number of musclefibres in a surface-unity much larger than is the case in the irradiated one. (Fig. 11 and 12.)

In the irradiated larva the musclelayer is underbroken by small cavities wherein no musclefibre is to be seen. Probably not all the medial myotomcells are differentiated into musclecells, or the musclecells, originally developed have degenerated and have disappeared.

The mesodermal tissue around the normal pronephros shows more vascularity than the corresponding tissue in the irradiated larva (photo 11 and 12). A section through the normal pronephros shows the lumina of the tubuli narrow and numerous, whilst in the section through the irradiated pronephros the lumina of the tubuli are wide and but few in number. As it is a fact, that during development the pronephrostubuli become narrower and narrower and take a more and more wrinkled course, it is obvious that the small number of wide lumina found in the sections through the pronephros of the irradiated larva, proves that the irradiated larva is at least in a younger stage of development. Examining the pronephrostubuli of the irradiated larva with a larger magnification, it appears that the wall of these tubuli have an irregular thickness, that only few cells are visible and many of the cells have degenerated in such a way, that from the nuclei only irregular lumps have been left, whilst the cellprotoplasm is vacuolized (photo 13 and 14).

Whereas the degeneration of the pronephros of the irradiated larva is not dubious we may conclude that daughtercells of irradiated mothercells have degenerated.

This same phenomenon offers the liver of the irradiated larva. The liver of the normal larva of 40 days is built up by liverbeams forming a network in which meshes bloodvessels are found (photo 16). The liver of the irradiated larva is much smaller than the normal one, moreover it consists partly of necrotic tissue, whilst in the more normal parts, the cells are

highly troubled (photo 15). However in the necroic part the liverstructure is to be recognized, so that we must conclude that in a younger stage the liverstructure has indeed been formed, but has soon degenerated.

Comparing the genital glands of the normal and irradiated larvae, the normal gland proves to be more than three times larger than that of the irradiated one. So the second irradiation experiment shows as well as the first experiment, that the development after irradiation is a retarded one. Besides the second proof, in which the animals lived 38 days after irradiation, demonstrates that the different organs have a volume much smaller than the normal larvae of the same age, or normal larvae in the same stage of development. For instance the optic vesicle and the heart of larva C III (40 days) are much smaller than these organs are in the normal larva of 40 days, but also smaller than are the optic vesicles and the heart of a normal larva of 17 days, having not yet passed the development stage of the irradiated (photo 23).

So it is probable that the irradiation caused the death and atrophy of a quantity of cells, the mothercells of which had been irradiated.

In the third place this experiment demonstrates, that irradiation, once only, in a very early stage of development, causes the degeneration of tissues, arisen from the irradiated mothercells: liver, pronephros, (photo 13 and 15). Retardation of development and degeneration of the tissues were the principal phenomena due to our irradiation experiments. The question arises, what is the relation between both these phenomena and the quantity as well as the quality of the administered X-rays?

III. *Irradiationexperiment.*

Four twigs with axolotleggs, irradiated one day after fertilization with respectively method I, II, III, and IV, were photographed 10 days after irradiation. (Fig. 17—22.) The eggs, irradiated with method I, scarcely differ from the normal eggs. The eggs H II compared with normal ones show much more difference, whilst the eggs H III and the eggs H IV show big retardation of development. Besides the eggs H IV are all troubled and did not develop at all.

IV. *Irradiation experiment.*

Some eggs were irradiated half a day after fertilization with respectively method I, II, III and IV. (Series A I, A II, A III and A IV.)

The eggs A IV (six in number) did not develop at all. 3 eggs were preserved 7 days, 3 eggs 18 days after irradiation. All these eggs were dead, in the yolkmass there was not any trace of nucleardivision or celllimitation. Besides, the preparations swarmed with bacili, so that we must conclude that the process of rotting had begun long before the preservation.

The eggs A III, A II and A I were preserved the nineteenth day. Comparing these series mutually, as well as with controlseries of eggs of 17 days, it appears that the changes due to irradiation are smallest in the

series of A I, greatest in the series of A III. For instance the optic vesicle of A III not yet being inverted is still in connection with the brainvesicle by a canal (photo 24), the lens has penetrated from the surface of the body through the mesenchym to the optic vesicle and is a simple vesicle, without the slightest differentiation.

The optic vesicles of A II and A I are much more differentiated (photo 25). A I as the best developed auricular vesicle, though there is not yet a division of this vesicle as is the case in the controllarve. The neuro-epitheliumcells along the wall of the vesicle are more or less developed in A I; in A II and A III they are scarely to be discerned besides, in A II and A III all the cells of the vesiclewall are troubled.

The epithelialcells of the oral cavity of A II and A III are also troubled, whilst there is an abnormal outgrowth of cells together with an insufficient developing of the tongue.

Whilst in normal control-larvae of 17 days the tongue is formed by invagination of the oral-epithelium, at both sides of the medial line, and the epithelium of the tongue is separated from a cartilaginous layer by mesodermal tissue (photo 27) there is an incomplete invagination of the mouth-epithelium in A III (photo 26), so that the tongue becomes flatter. The cell-outgrowth under the epithelium proves to be a conglomerate of epitheliumcells and mesenchymalcells both rich in yolk-particles. A II has nearly the same anomalies as A III. A I possesses a normal tongue.

The heartcells of A II and A III are all in a degenerative state, though in A II there is but a slight degeneration, in A III a very strong one. Sections through the heart of A I were destroyed, so that only the hearts of A II (photo 29) and of A III (photo 30) can be compared with normal controlhearts (photo 28). The photos show, that the normal heart is more than twice as great as the hearts of A II and A III. The normal heart is filled up with bloodcells, whilst A II has but few bloodcells and A III has not got them at all. The hearts of A II and A III by no way fill up the body-cavity, whilst in the normal larva, there is but a small slit between the heart wall and the mesoderm. Persecuting the sections of the larvae distally, the difference between normal and X-rayed larvae becomes greater, e.g. the liver of A I is filled with yolkmaterial whilst the meshwork of liverbeams is much coarser than that of the normal larva (photo 31). A II and A III scarcely show any liverstructure (photo 32, 33 and 34), there are only few tubuli in the midst of a dense yolkmass on one side, whilst the other side only consists of necrotic tissue. There are no bloodcells found in the livers of A II and A III.

The pronephros shows in A I some wide tubuli, embedded in mesodermal tissue, which has scarcely any cells visible. A section through the pronephros of A II shows but few lumina of the tubuli, whilst the walls of them are formed by picnotic and highly troubled cells. A III possesses still much wider tubuli, whilst the mesodermal tissue around them is totally failing.

The irradiation experiments 3 and 4 show macroscopically as well as

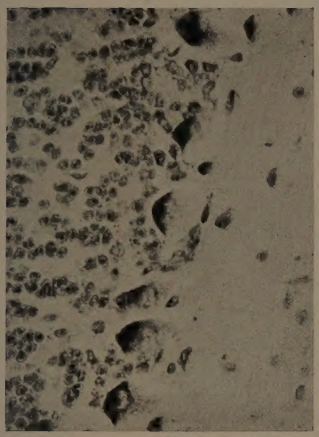

Fig. 1. PURKINJE-cells of X-rayed cavia, chromatine is disks-shaped.

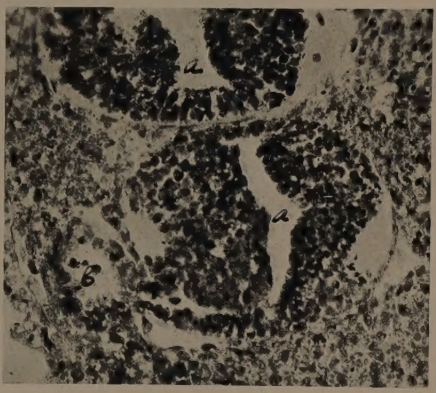

Fig. 3. Section through axolotl B III X-rayed $1^1/_2$ day after fertilization, preserved the 9th day. Objective 20. Ocular 5.

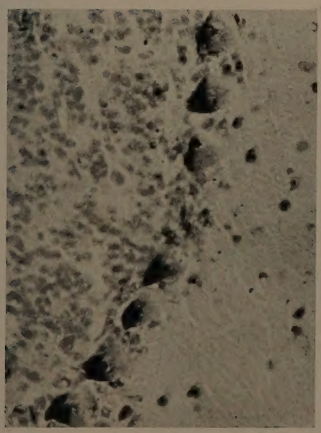

Fig. 2. PURKINJE-cells of normal cavia, with disk-shaped chromatine.

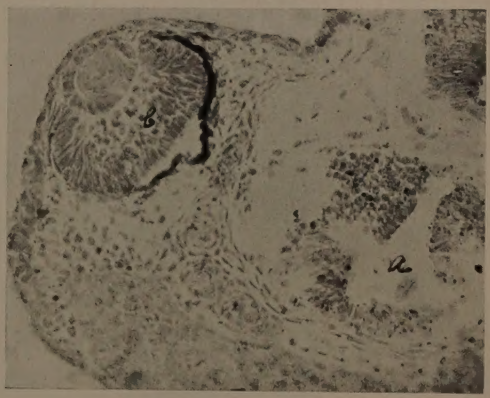

Fig. 4. Section through normal axolotl larva of 9 days. Objective 20. Ocular 5.

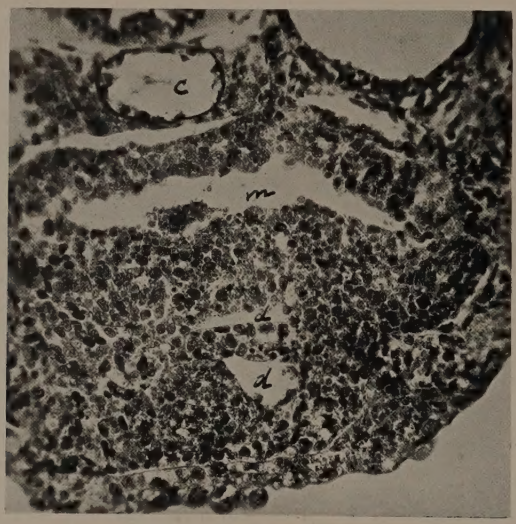

Fig. 5. Section through axolotl B III X-rayed $1^1/_2$ day after fertilization, preserved the 9th day. Objective 20. Ocular 5.
c = notochord m = oral cavity d = heart.

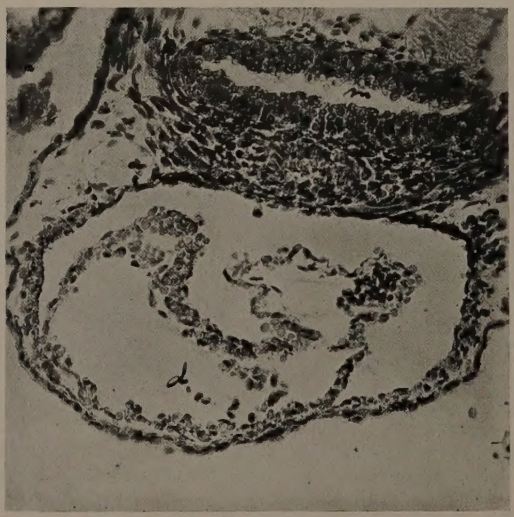

Fig. 6. Section through normal axolotl of 9 days. Obj. 20. Ocul. 5.
m = oral cavity. d = heart.

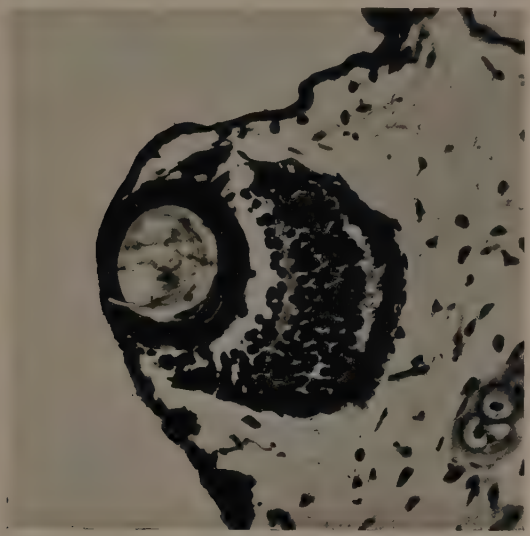

Fig. 7. Section through the eye of axolotl C III, X-rayed $2^1/_2$ days after fertilization, preserved the 40th day. Objective 20. Ocular 5.

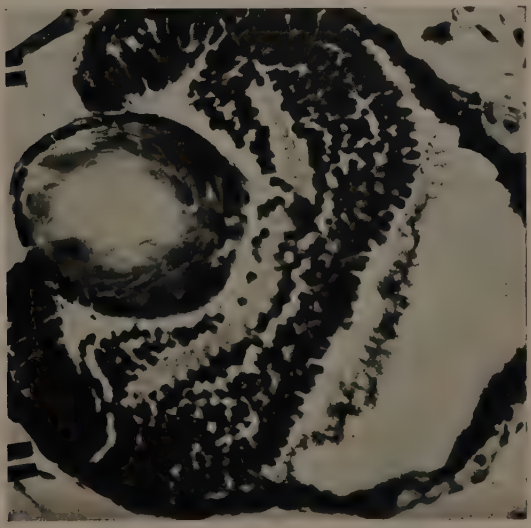

Fig. 8. Section through the eye of a normal axalotl of 40 days. Objective 20. Ocular 5.

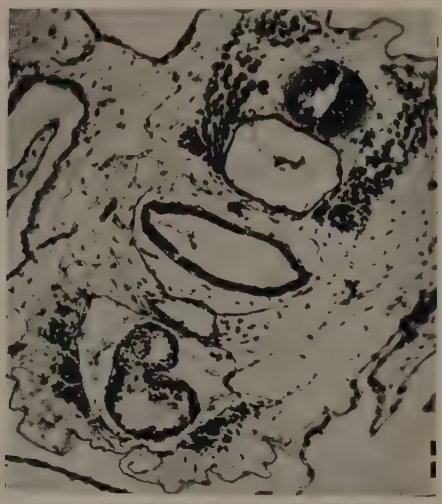

Fig. 9. Section through the heart of axolotl C III X-rayed $2^1/_2$ day after fertilization, preserved the 40th day. Objective 10. Ocular 5.

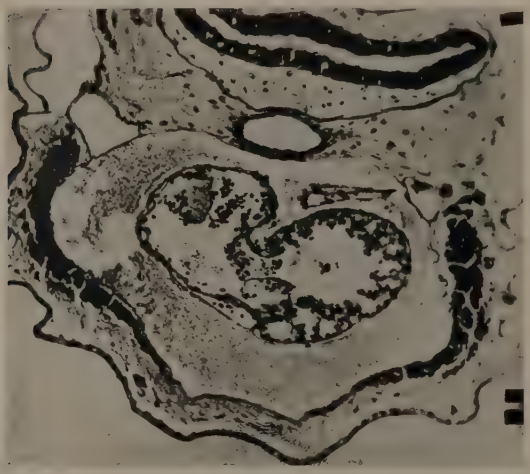

Fig. 10. Section through the heart of a normal axolotl of 40 days. Objective 10. Ocular 5.

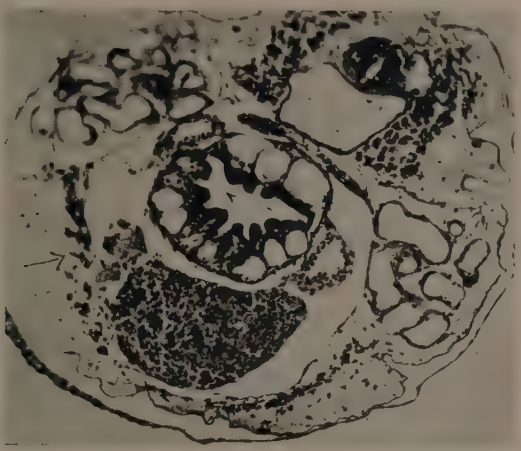

Fig. 11. Section through the pronephros of axolotl C III, X-rayed $2^1/_2$ days after fertilization preserved the 40th day. Objective 10. Ocular 5.

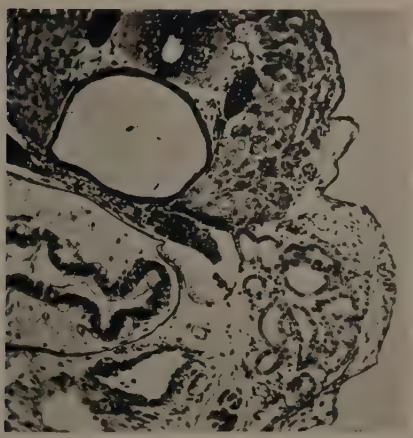

Fig. 12. Section through the pronephros of a normal axolotl of 40 days. Objective 10. Ocular 5.

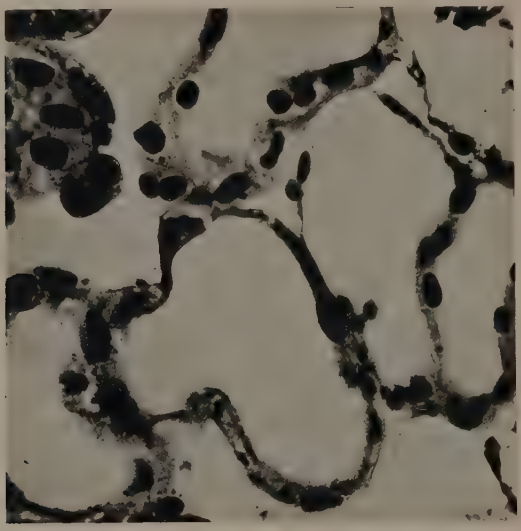

Fig. 13. Pronephros tubuli of axolotl C III.

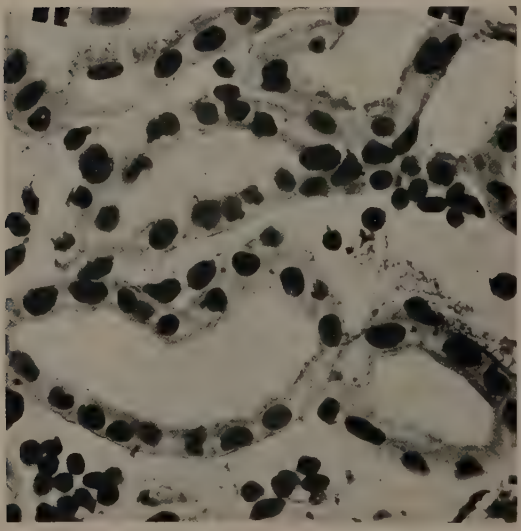

Fig. 14. Pronephros tubuli of a normal axolotl of 40 days.

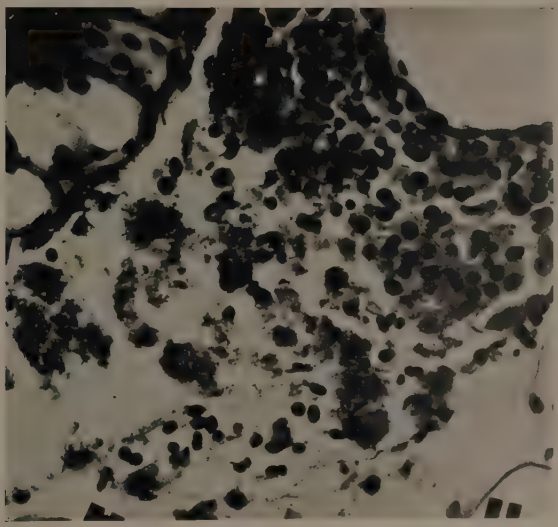

Fig. 15. Section through the liver of axolotl C III. X-rayed $2^1/_2$ days after fertilization, preserved the 40th day. Objective 20. Ocular 10.

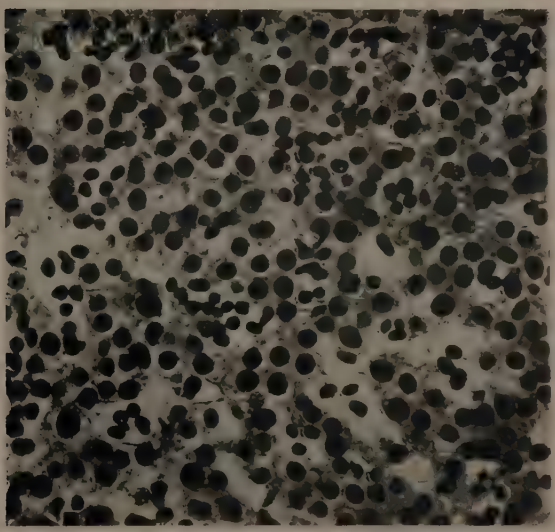

Fig. 16. Section through the normal liver of an axolotl ot 40 days. Objective 20. Ocular 10.

Fig. 17. Normal axolotl larvae ± 10 days old.

Fig. 18. Normal axolotl larvae ± 10 days old.

Fig. 19. Axolotl larva H I, X-rayed 1 day after fertilization with method I. 10 days old.

Fig. 20. Larvae H II, X-rayed 1 day after fertilization with method II.

Fig. 21. Larvae H III, X-rayed with method III.

Fig. 22. Larvae H IV, X-rayed with method IV

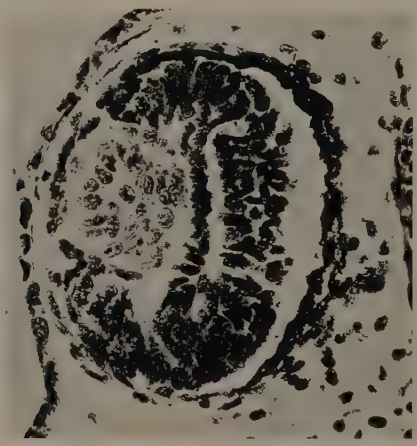

Fig. 23. Section through the normal eye of an axolotl of 17 days. Objective 20. Ocular 5.

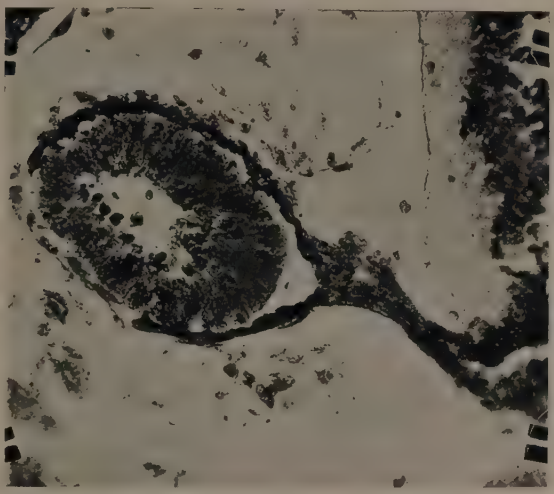

Fig. 24. Section through the eye of an axolotl X-rayed $^1/_2$ day after fertilization (A III) preserved the 19th day. Objective 20. Ocular 5.

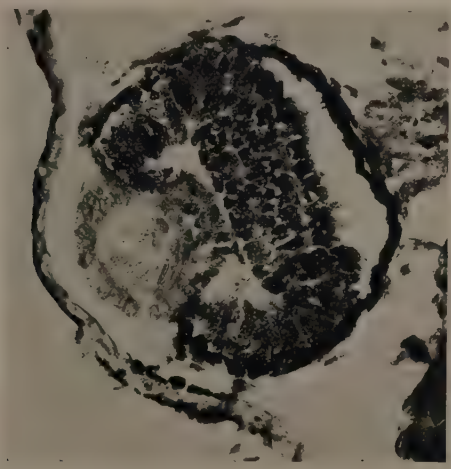

Fig. 25. Section through the eye of an axolotl X-rayed $^1/_2$ a day after fertilization A II preserved the 19th day.

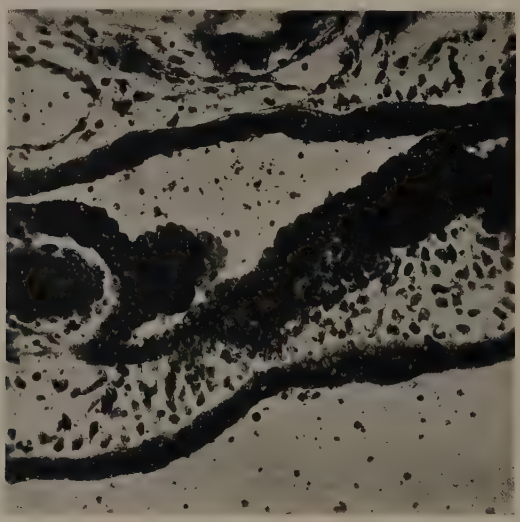

Fig. 26. Section through the tongue of an axolotl A III, X-rayed half a day after fertilization and preserved the 19th day.

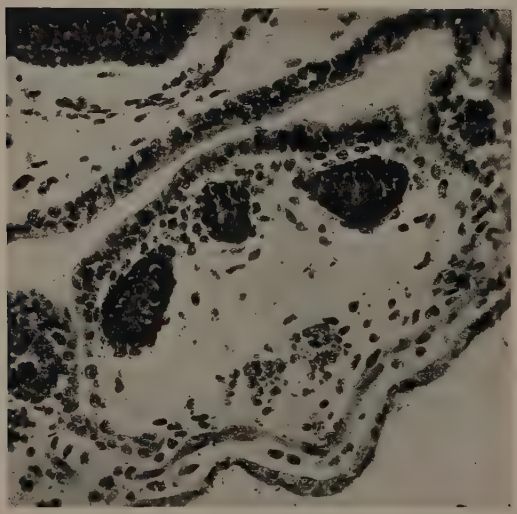

Fig. 27. Section through the tongue of a normal axolotl of 17 days.

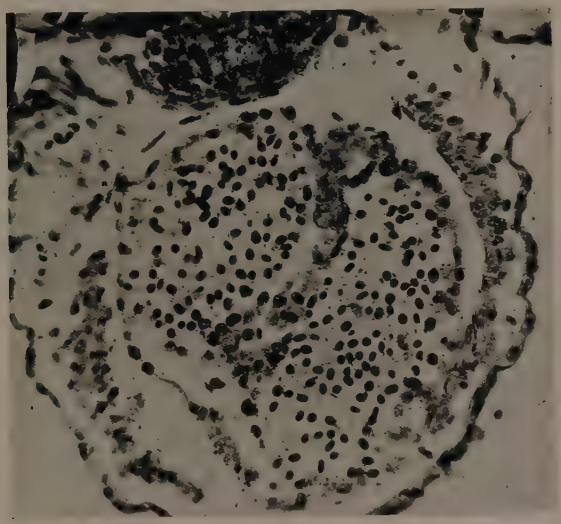

Fig. 28. Section through the heart of a normal axolotl of 17 days.

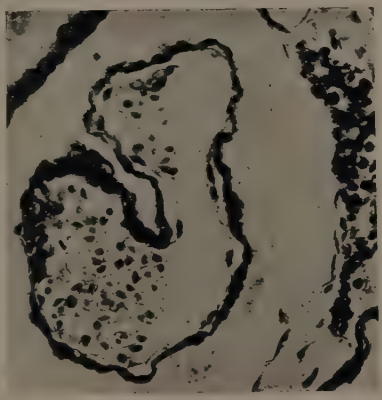

Fig. 29. Heart of axalotl A II.

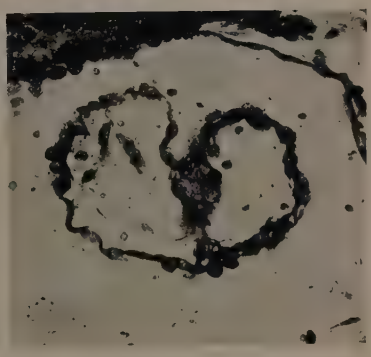

Fig. 30. Heart of axolotl A III

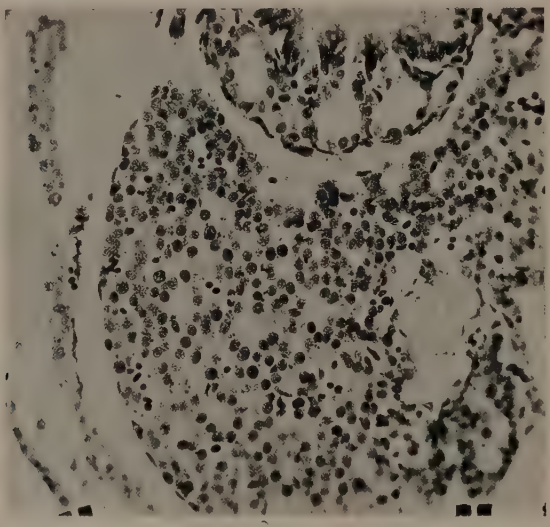

Fig. 31. Section through the liver of a normal axolotl of 17 days.

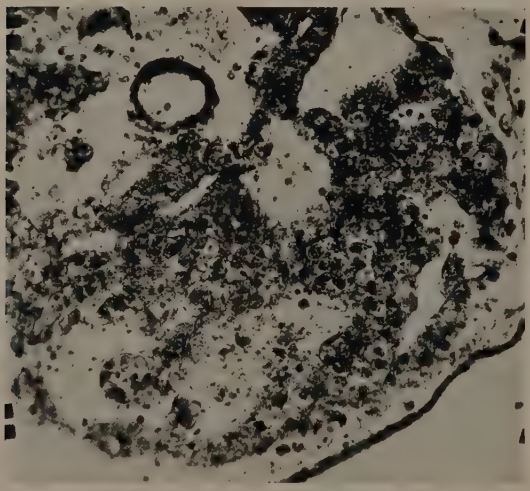

Fig. 32. Section through the liver of axolotl A II, preserved the 19th day.

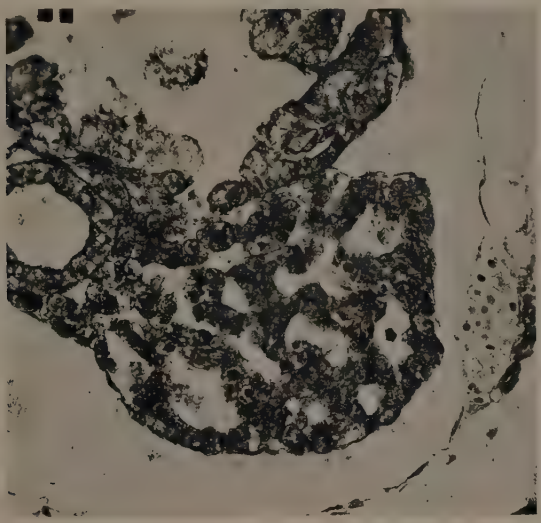

Fig. 33. Section through the liver of axolotl A III preserved the 19th day.

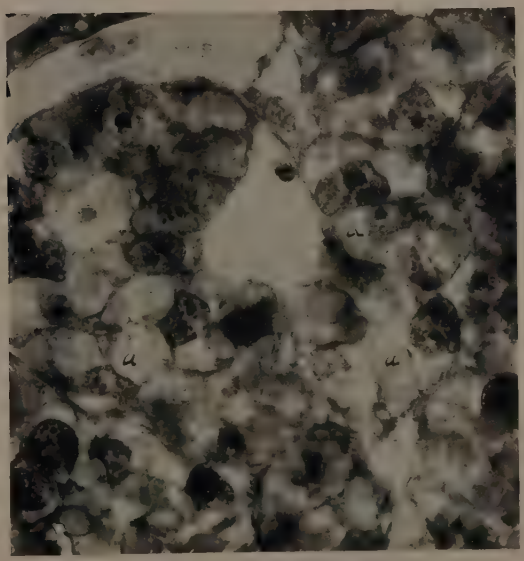

Fig. 34. As fig. 33 oil immersion. a = yolk particles.

Fig. 35. Optic vesicle of E III X-rayed $6^1/_2$ day after fertilization preserved the 19th day to compare with fig. 24. Objective 10. Ocular 5.

Fig. 36. Optic vesicle of E IV preserved the 19th day.

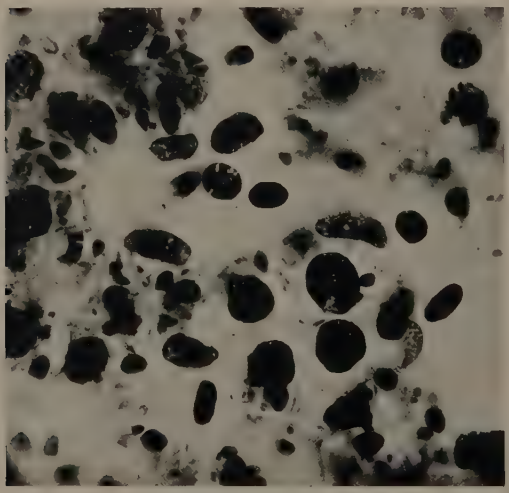

Fig. 37. Liver tissue of E III to compare with fig. 34.

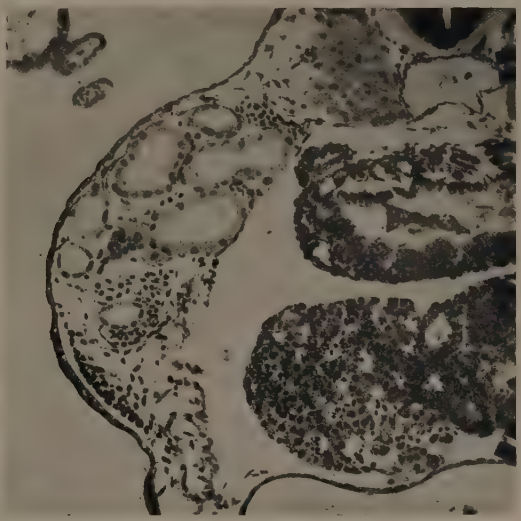

Fig. 38. Section through pronephros and liver of E III preversed the 19th day.

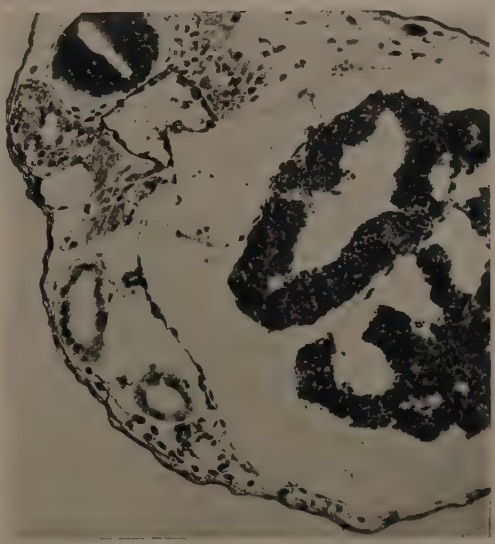

Fig. 39. Section through pronephros and liver of E IV preserved the 19th day.

microscopically, that the degree of retardation and degeneration increases from method I to method IV. The method II and III do not give such strong differences, although method III causes more degeneration in the tissue than method II. In reference to the question posed on pag. 238, the latter irradiation experiments prove, that, although the ionometric dose of A I (90 R) and A III (95 R) as well as the ionometric dose of A II (300 R) and A IV (285 R were nearly the same, the results, obtained with soft rays method I and II) are different from those obtained with hard rays (method III and IV), e.g., whilst A II more or less developed after irradiation, all the eggs of A IV died within a week after irradiation.

A second factor, wherupon the degree of retardation and degeneration depends, is the age at which the irradiation of the egg takes place. The younger the egg, so much the greater is the retardation and so much the more intensive is the degeneration of the different tissues, which fact is proved by the :

V. *Irradiation experiment.*

Some eggs were irradiated 6½ days after fertilization (series E I, E II, E III, and E IV) and were preserved the 19th day. These series have been compared with the series A I, A II, A III, and A IV, irradiated ½ a day after fertilization, and also preserved the 19th day.

Comparison of these series gave the following differences :

The optic vesicle of E I is much better differentiated than is the case with the optic vesicle of A I. Auricular vesicles of A I and E I show scarcely any differences. The liver and ventral wall of the digestive tract of A I are filled up with yolkmass, whilst in E I the yolk has totally disappeared.

The pronephros of E I is embedded in mesodermal tissue rich in blood-vessels, the pronephros-tubuli are rather narrow, whilst A I possesses much wider tubuli, embedded in mesodermal tissue not so rich in bloodvessels.

The differences between A II and E II are still more distinct. E II possesses already a ductus endolymphaticus more or less differentiated and neuro-epitheliumcells along the wall of the auricular vesicle. In A II these differentiations are missing.

The pronephros tubuli of E II are formed by well-shaped epithelial-cells, whilst these cells in the pronephros of A II are irregularly shaped and have for the greater part fallen asunder in a grainy mass. The nuclei, which are yet recognizable show a loss of chromatine substance, whilst the walls of the tubuli are filled up with yolkmass, which material in the pronephros of E II has totally disappeared. The liver of E II built up by distinct liverbeams, surrounding the bloodvessels, is greatly different from the liver of A II which has almost totally degenerated.

The differences between A III and E III are still greater, as show the optic-vesicle, (photo 35) the tongue, the liver (photo 37) and the pronephros (photo 38).

Whilst none of the A IV eggs developed, the E IV eggs lived at least

19 days. They possess a pair of scarcely developed optic-vesicles (photo 36), a much degenerated liver- and pronephros-tissue, (photo 39) having nearly the same aspect as the tissues of A III.

Notwithstanding the E series being always better differentiated than the A series, yet when comparing the E series mutually, we see that there is a big difference between E I, E II, E III and E IV, in the same line as is the case with the different A series.

So, comparison of the E series, and A series shows, that the influence of the X-rays is smaller when the irradiation takes place at a later stage of development of the fertilized egg. As far as the retardation is concerned it is obvious, that, if X-raying generally causes retardation, this retardation will be less, when the X-raying takes place after the first stages of swift celldivisions being finished. A larva, the development of which is retarded by being X-rayed the first day of development (A series), will not be able to pass through as great a number of celldivisions as a larva that was X-rayed the 6th day of development (E series).

The A series shows more degeneration than the E series, this may be explained by the hypothesis, that after X-raying a definite cell, the successive daughtercells degenerate more and more. As, after 19 days the daughtercells of an A series have a more distant relation to the X-rayed cells themselves (18 days of celldivision), than the daughtercells of the E series (12 days of cell-division after irradiation), the degeneration of the A series must be greater than the degeneration of the E series.

Supposing this hypothesis to be right, then the degeneration of the E series ought to appear at a later stage.

Examining however an E axolotl after 40 days, (an age, that was generally not reached by the A exolotls), and comparing this axolotl with a normal one, it appears, that instead of a more degenerative state a distinct regeneration has taken place, so that the difference between the X-rayed tissues and the normal ones are but slight. As this fact proves against the supposition above mentioned, the question arises, in how far the more or less embryonic and undifferentiated character of the cells that are to be X-rayed, plays a part?

So the hypothesis may be posed, that the less differentiated and the more embryonic a cell is, the greater is the influence of X-rays upon the progeniture of the X-rayed cell.

The differences between the A series and E series plead in favour of this hypothesis e.g. the axolotls A IV did not develop at all, whilst the E IV larvae reached the ultimate age of respectively 19, 20, and 23 days, though they were in a very deplorable condition and showed highly degenerated organs.

The A III series died within 25 days, except one that lived 62 days, but so much retarded in development that it could not be put on one line with an E III series of 40 days. The E III-axolotls however regenerated. Two larvae of this series, though showing retardation in the beginning of their

development, behaved afterwards as if they were not irradiated at all and were preserved at the 98th days. Anatomically, there was no difference between these axolotls and normal ones of the same age; however our preparations did not permit the detecting of fine histological differences which might be present.

Regeneration of the A-axolotls only occurred when the irradiation had been very slight (A. I).

The following state too pleads for the secondly mentioned hypothesis. 98 days after the fertilization of the eggs, which were X-rayed, though at different stages, the following collection was left:

Method	I.	II.	III.	IV.
A series (X-raying, after ½ day)	1	0	0	0
B series (X-raying, after 1½ days)	1	0	0	0
C series (X-raying, after 2½ days)	3	1	0	0
D series (X-raying, after 4½ days)	2	1	1	0
E series (X-raying, after 6½ days)	3	2	2	0

This small table points in the direction, that axolotleggs, X-rayed at a later stage, are more able to endure the influences of the X-raying, than those, X-rayed directly after fertilization.

The axolotls, which had reached the 98th day, showed no or but very slight differences with the normal ones.

From the many treatises on the influences of X-rays on developing forms. those of GILMAN and BAETJER are related to ours. However, the experiments of the authors differed from ours, in so far, that they exposed the eggs of amblystoma daily 15 minutes to the X-rays and the periods of this treatment were different. The ionometric dose and the kind of administered X-rays have not been mentioned. The results were, that, during the first days, there was an accelerating of development, after 3 or 4 days the development became abnormal and soon retardation appeared. If the period of daily exposures had been a short one, a restitution occurred. Deviations of development were: bad development of the eyes, deformity of parts of the mouth and nondevelopment of the external gills.

Deformity of the mouthregion, as well as bad development of the eyes, could also be stated by us. As our animals were not preserved before the 8th day of development, we do not know if during the first days an acceleration of development took place. However, as already the 8th day the development proved to be highly retarded, we doubt if an acceleration would ever have taken place. The different results are perhaps due to a difference in the quality and quantity of the rays, or the modus of administering the rays.

The experiments of BARDEEN and others, which authors fertilized the eggs of tadpole with X-rayed spermatozoa, differ too much from ours. than that comparison of the results would be necessary.

Comparing our experiments with experiments on the influence of X-rays upon malignant growths, we can state many analogous results.

I. CONTAMIN proved that the younger the malignant growth is, the stronger is the influence of X-rays upon the growth.

II. NOGIER, JAUBERT DE BEAUJEU and CONTAMIN proved that the X-rays rather hindered the growth of an X-rayed and afterwards inoculated adeno-carcinoma, than that they prevented them from taking on inoculation.

MARIE, CLUNET and RAULOT—LAPOINTE stated that tumourcells altered their characters by the influence of X-rays. When, after a time, the X-raying-treatment was stopped, the newly obtained characters disappeared, whilst there was a tendency to regain the original normal characters.

NOGIER and RAGOUD pointed out, that, if malignant growths were X-rayed at intervals, there was a gradual diminution of the X-ray-effects viz. the successive cell-generations underwent less and less the influence of the X-raying.

Principally these observations on tumourcells agree with the observations we made in the developing tissue of the axolotl-larva.

A. Diminution of growth.

B. Increase of degenerative characters in the daughtercells.

C. Gradual loss of sensibility for the X-rays in proportion to the generations of the cells.

Resuming the results of our experiments, we found:

1. Experiments with young caviae did not give any results.

2. The displacing of the chromatine in the PURKINJE-cells of the cerebellum after radium-irradiation, described by MOROWOKA and MOTT is valued by us as a phenomenon, that may appear in normal animals as well and may be due to technical inexactitudes.

3. X-raying, one time, of fertilized axolotleggs caused:

A. A retardation of the development of the larva.

B. A diminution of volume of the developing organs, not only in proportion to normal control animals of the same age, but also compared with normal younger larvae, the development of which is in the same stage as that of the irradiated ones.

C. A degeneration of the daughtercells of irradated mothercells.

4. The degree of retardation and degeneration depends on the age at which the developing egg is X-rayed, viz. the nearer the X-rayed cells stand in generation to the fertilized egg, the greater is the influence of the X-rays upon their progeniture.

5. Regeneration only appears, when the quantity of administered X-rays is very slight, or when the X-raying takes place at not too early an age.

6. With restriction to our data it appears, that, when the ionometric dose is the same, the damage, caused by hard rays, is different from the damage, caused by soft rays.

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Mathematics. — *Zur Theorie der alternierenden Tensoren* a_{ikl}. Von R. WEITZENBÖCK.

(Communicated at the meeting of March 23, 1929).

In einem linearen Gebiete G_n von n-ter Stufe, also bei n Veränderlichen x_i $(n \geqq 4)$ heisst ein alternierender Tensor zweiter Stufe a_{ik} *einfach*, wenn es zwei Punkte x und y gibt, sodass

$$a_{ik} = (xy)_{ik} = x_i\ y_k - x_k\ y_i$$

für alle i und k gilt. Es besteht dann der bekannte Satz, dass sich jeder Tensor a_{ik} durch höchstens $\left[\frac{n}{2}\right]$ $\left(= \text{grösstes Ganzes von } \frac{n}{2}\right)$ einfache Tensoren $p_{ik}, q_{ik}, \ldots, r_{ik}$ additif darstellen lässt:

$$a_{ik} = p_{ik} + q_{ik} + \ldots + r_{ik}.$$

Ausserdem gibt es für jedes n auch Tensoren a_{ik}, bei denen diese Anzahl $\left[\frac{n}{2}\right]$ der Komponenten $p_{ik}, q_{ik}, \ldots$ nicht verringert werden kann.

Das entsprechende Problem für alternierende Tensoren dritter Stufe a_{ikl} ist bisher nur für die niedrigsten Werte von n bis zu $n = 7$ gelöst. Wir geben im Folgenden eine obere Grenze für die Anzahl $a_n^{(3)}$ der einfachen Tensoren $p_{ikl}, q_{ikl}, \ldots$, die nötig sind um einen allgemeinen alternierenden Tensor a_{ikl} in der Gestalt

$$a_{ikl} = p_{ikl} + q_{ikl} + \ldots + r_{ikl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

darstellen zu können. Für die niedrigsten Werte von n wird diese Grenze stets erreicht; dass dies bei jedem n der Fall sein kann, konnte ich bisher nicht beweisen.

Es ist:

$$a_n^{(3)} \leqq \left[\frac{n^2 - 6n + 13}{4}\right] = \left[\left(\frac{n-3}{2}\right)^2 + 1\right] \quad . \quad . \quad . \quad . \quad (2)$$

Zum Beweise von (2) bemerken wir vorerst, dass die durch die Darstellung (1) gestellte Frage ersetzt werden kann durch diese: Im G_n sind beliebig viele einfache Tensoren (Ebenen)

$$\alpha_{ikl}^{(\nu)} = (x^{(\nu)}\ y^{(\nu)}\ z^{(\nu)})_{ikl} \quad , \quad (\nu = 1, 2, 3, \ldots) \quad . \quad . \quad . \quad . \quad (3)$$

gegeben; aus ihnen wird der Tensor

$$a_{ikl} = \alpha_{ikl}^{(1)} + \alpha_{ikl}^{(2)} + \ldots \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

gebildet und für diesen wird eine Darstellung (1) gesucht mit möglichst wenig Komponenten $p_{ikl}, q_{ikl}, \ldots, r_{ikl}$.

Wir wählen im G_n ein $G_{n-1}\, v'$ und einen Punkt s, der nicht in v' liegt, sodass $(v'\, s) = v^i s_i \neq 0$ ist. Von s aus projizieren wir alle Punkte $x^{(\nu)}$, $y^{(\nu)}$ und $z^{(\nu)}$ auf v' und erhalten die Punkte $\bar{x}^{(\nu)}$, $\bar{y}^{(\nu)}$, $\bar{z}^{(\nu)}$. Dann gilt

$$\begin{cases} x^{(\nu)} = \alpha^{(\nu)} \,.\, s + a^{(\nu)} \,.\, \bar{x}^{(\nu)} \\ y^{(\nu)} = \beta^{(\nu)} \,.\, s + b^{(\nu)} \,.\, \bar{y}^{(\nu)} \\ z^{(\nu)} = \gamma^{(\nu)} \,.\, s + c^{(\nu)} \,.\, \bar{z}^{(\nu)} \end{cases}$$

und daher nach (3):

$$\alpha^{(\nu)}_{ikl} = a^{(\nu)}\, b^{(\nu)}\, c^{(\nu)} (\bar{x}^{(\nu)}\, \bar{y}^{(\nu)}\, \bar{z}^{(\nu)})_{ikl} + \sum_1^3 b^{(\nu)}\, c^{(c)}\, \alpha^{(\nu)} (\bar{y}^{(\nu)}\, \bar{z}^{(\nu)}\, s)_{ikl} \quad . \quad . \quad (5)$$

Dies kann kurz so ausgedrückt werden: Jede Ebene $\alpha^{(\nu)}_{ikl}$ des G_n wird dargestellt als Summe einer im Gebiete $G_{n-1}\, v'$ liegende Ebene

$$p^{(\nu)}_{ikl} = (\bar{x}^{(\nu)}\, \bar{y}^{(\nu)}\, z^{(\nu)})_{ikl}$$

und dreier durch den Punkt s gehenden Ebenen σ_{ikl}. Nach (4) und (5) wird demnach:

$$a_{ikl} = \sum_\nu p^{(\nu)}_{ikl} + \sum_\tau \sigma^{(\tau)}_{ikl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hier führt $\sum_\nu p^{(\nu)}_{ikl}$ auf höchstens $a^{(3)}_{n-1}$ Komponenten, $\sum \sigma_{ikl}$ dagegen auf höchstens $a^{(2)}_{n-1} = \left[\frac{n-1}{2}\right]$ Komponenten, d.h. aus (6) folgt:

$$a^{(3)}_n \leqq a^{(3)}_{n-1} + a^{(2)}_{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Wir zeigen jetzt noch, dass man die rechte Seite dieser Ungleichung noch um Eins verminderen kann, dass also auch

$$a^{(3)}_n \leqq a^{(3)}_{n-1} + a^{(2)}_{n-1} - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

gilt, woraus sich dann mühelos die Formel (2) ergibt.

Die Ebenen $o^{(\tau)}_{ikl}$ durch s schneiden das $G_{n-1}\, v'$ nach Geraden $\pi^{(\tau)}_{kl}$, sodass in (6) die zweite Summe rechts ersetzt werden kann durch $[s_i, \Sigma\, \pi^{(\tau)}_{kl}]$. Hier wird $\Sigma\, \pi^{(\tau)}_{kl}$ ersetzbar durch $\sum_1^\varrho \omega^{(\lambda)}_{kl}$, wo $\omega^{(\lambda)}_{kl} = (\xi^{(\lambda)}\, \eta^{(\lambda)})_{kl} = \xi^{(\lambda)}_k\, \eta^{(\lambda)}_l - \xi^{(\cdot)}_l\, \eta^{(\cdot)}_k$ eine Gerade in v' vorstellt und $\varrho < \left[\frac{n-1}{2}\right]$ ist.

Gilt hier das Zeichen $<$, so ist (8) bewiesen. Sei also $\varrho = \left[\frac{n-1}{2}\right]$; dann müssen wir zwei Fälle unterscheiden, je nachdem $n-1$ gerade oder ungerade ist. Bei $n-1 = 2m$ ist im $G_{n-1}\, v'$ der Tensor K

$$\omega_{ik} = \sum_{\lambda=1}^{\varrho} \omega^{(\lambda)}_{ik} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

und sein hiezu konjugierter $K' = [\omega_{12} \,.\, \omega_{34} \,.\,.\,.\,.\, \omega_{2m-3,\,2m-2}]$ nicht-singulär und wenigstens eine Komponente $\omega_{ik}^{(\lambda)}$ von (9) kann in einer der Ebenen $q_{ikl}^{(\mu)}$ von

$$a_{ikl} = \Sigma\, p_{ikl}^{(\nu)} + \Sigma\, \sigma_{ikl}^{(\tau)} = \Sigma\, q_{ikl}^{(\mu)} + \Sigma\, [s_i \,,\, \omega_{kl}^{(\lambda)}] \quad . \;.\;. \quad (10)$$

gewählt werden, sodass in der rechten Seite dieser Gleichung die Summe

$$q_{ikl}^{(\mu)} + [s_i \,,\, \omega_{kl}^{(\lambda)}] \quad . \;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (11)$$

durch eine einzige Ebene p_{ikl} ersetzbar wird.

Ist aber $n - 1 = 2m - 1$, also ungerade, dann wird $\varrho = m$ und ω_{ik} von (9) bestimmt in v' ein Brenngebiet G_{n-2}, dem alle Komponenten $\omega_{ik}^{(\lambda)}$ angehören. Dieses G_{n-2} schneidet wenigstens eine der Ebenen $q_{ikl}^{(\mu)}$ von (10) längs einer Geraden, die wir als Komponente $\omega_{ik}^{(\lambda)}$ wählen können, wodurch wir wieder bei (11) angelangt sind.

Wir bemerken schliesslich noch, dass man diese Uberlegungen auch allgemein, ohne von der Darstellung (4) Gebrauch zu machen, anstellen kann. Man hat dann auszugehen von der für jedes a_{ikl} giltigen Identität

$$(\pi^{n-3}\, a^3)\,(s\,v') \equiv -\,(n-3)\,(\pi^{n-4}\, a^3\, s)\,(\pi\, v') + 3\,(\pi^{n-3}\, a^2\, s)\,(a\, v'), \quad . \quad (12)$$

die hier die Rolle der Gleichung (6) übernimmt.

Mathematics. — *Eine Kongruenz von elliptischen Raumkurven vierter Ordnung.* Von Prof. JAN DE VRIES.

(Communicated at the meeting of March 23, 1929.)

1. Die elliptischen Raumkurven ε^4 durch fünf Punkte B_k, welche die Geraden c_1 und c_2 je zweimal treffen, bilden eine Kongruenz E, welche durch zwei Büschel (Q_1) und (Q_2) von quadratischen Flächen erzeugt werden kann. Die Basis von (Q_1) besteht aus c_1 und der kubischen Kurve γ_1^3 durch B_k, welche c_1 zweimal schneidet; analog gehen die Flächen Q_2 durch c_2 und die Kurve γ_2^3.

Jede Q_1 enthält ein Büschel (ε^4) mit sieben Basispunkten; (Q_1) bestimmt auf c_2 eine Involution von Basispunkten C'_2, C_2''.

Auf einer Geraden g bestimmen (Q_1) und (Q_2) i. A. zwei I^2, welche i. A. *ein* Punktepaar gemein haben, wonach g Bisekante für *eine* ε^4 ist. $(\mathsf{P}^5\,\mathsf{B}^3 = 1)$.

2. Ordnet man zwei Flächen Q_1, Q_2 einander zu, wenn sie sich auf der Geraden l treffen, so ergibt sich zwischen (Q_1) und (Q_2) eine Verwandtschaft (2,2). Demnach sind auch die Kegelschnittbüschel (K_1) und (K_2), welche (Q_1) und (Q_2) in der Ebene β_{123} $(B_1 B_2 B_3)$ bestimmen, in einer (2,2) begriffen. Mit Hülfe einer quadratischen Transformation, mit Hauptpunkten B_1, B_2, B_3, ersieht man, dass (K_1) und (K_2) eine Kurve λ^8 erzeugen, welche *vierfache* Punkte in B_1, B_2, B_3 und *Doppelpunkte* in den Spuren C_1 und C_2 von c_1, c_2 hat; ausserdem hat diese rationale Kurve noch einen Doppelpunkt in der Spur der ε^4, welche l zweimal trifft.

Betrachtet man als *Bild* einer ε^4 den Punkt E, welchen sie ausser B_1, B_2, B_3 mit β_{123} gemein hat, so entspricht dem System Λ der ε^4, welche l treffen, demnach eine *Bildkurve* $\lambda^8\,(B_1^4\,B_2^4\,B_3^4\,C_1^2\,C_2^2)$.

Weil zwei Kurven λ^8 acht Punkte E gemein haben, ist der *Ort* jenes Systems eine *Fläche* Λ^8, und gibt es acht Kurven in E, die sich auf zwei Geraden stützen $(\mathsf{P}^5\,\mathsf{B}^2\,\nu^2 = 8)$.

Die ε^4, welche γ_1^3 in einem festen Punkt treffen, liegen auf der durch diesen Punkt gelegten Fläche Q_2. Demnach sind γ_1^3 und γ_2^3 *Doppelkurven* von Λ^8; auf dieser Fläche sind B_k *vierfache Punkte* c_1 und c_2 *Doppelgeraden*.

3. *a*. Jede der 5 Kurven ϱ^3 durch B_k, welche sich auf c_1 und c_2 stützen, wird zu einer ε^4 ergänzt durch eine Bisekante, welche c_1 und c_1 trifft. (5 Figuren).

b. Die Transversale t_k durch B_k über c_1 und c_2 wird zu einer ε^4 ergänzt

durch eine ϱ^3, welche die übrigen Punkte B enthält, c_1 und c_2 schneidet und t_k zweimal trifft. (5 Figuren).

c. Jede Gerade $B_k B_l$ ist Bisekante einer ϱ^3 durch die übrigen Punkte B, welche auch c_1 und c_2 zweimal schneidet. (10 Figuren).

d. Der Kegelschnitt durch $B_1B_2B_3C_1C_2$ wird zu einer ε^4 ergänzt durch einen Kegelschnitt der B_4, B_5 enthält, jenen zweimal trifft und sich auf c_1, c_2 stützt. (5 Figuren).

4. Weil jede Gerade durch den Punkt M i. A. Sehne *einer* ε^4 ist, liegen die Stützpunkte der ε^4 auf einer Fläche μ^5, mit *dreifachem* Punkte M. Diese Fläche hat mit dem kubischen Kegel, welcher die durch M gelegte Kurve ε_0^4 projiziert, zunächst diese Kurve gemein; der Restschnitt besteht aus 11 Geraden, welche offenbar Bisekanten von ∞^1 Kurven ε^4 sind. Zu diesen *singulären Bisekanten* gehören die 5 Geraden MB_k und die 4 Geraden der durch M gelegten Flachen Q_1 und Q_2, welche sich in M treffen. Durch M gehen somit zwei Geraden s welche durch die Büschel (Q_1) und (Q_2) in derselben Involution geschnitten werden. Die Kurven ε^4, welche eine Gerade MB_k oder eine Gerade s treffen, bilden je eine Fläche F^4; für die übrigen 4 singulären Bisekanten ist die analoge Fläche eine Q_1 oder eine Q_2.

5. In einer Ebene φ bestimmen (Q_1) und (Q_2) zwei Büschel (k_1) und (k_2). Auf einer k_1 bilden die ε^4 eine Involution I^4; demnach enthält eine Q_1 sechs ε^4, welche φ berühren. Das System Φ der ε^4, welche φ berühren ordnet somit die Büschel (K_1) und (K_2), (§ 2), in eine Verwandtschaft (6,6). Demnach ist die *Bildkurve* des Systems Φ eine φ^{24} $(B_1{}^{12}B_2{}^{12}B_3{}^{12}C_1^6C_2^6)$. Mit einer Kurve λ^8 hat sie 24 Punkte E gemein; daher liegt das System auf einer Fläche Φ^{24}, mit *zwölffachen* Punkten B, *sechsfachen* Geraden c_1, c_2 und *sechsfachen* Kurven γ_1^3, γ_2^3. $(\mathsf{P}^5\,\mathsf{B}^2\,\nu_\varrho = 24)$.

Weil zwei Kurven φ^{24} sich in 72 Punkten E durchsetzen, ist $\mathsf{P}^5\,\mathsf{B}^2\,\varrho^2 = 72$.

6. Der Ort der Punkte in denen sich zwei Kurven der Büschel (k_1), (k_2) berühren, ist eine Kurve φ^5; sie ist offenbar der Ort der Punkte in denen die Ebene φ von Kurven ε^4 berührt wird.

Ausser der zweimal gelegten Kurve φ^5 hat φ mit der Fläche Φ^{24} noch eine Kurve φ^{14} gemein; diese ist der Ort der Punktepaare, welche die ε^4 des Systems Φ ausser ihren Berührungspunkten noch mit φ gemein haben.

Durch eine quadratische Transformation wird φ^5 in eine Kurve ψ^7 übergeführt, welche der Ort ist der Berührungspunkte der rationalen biquadratischen Kurven eines Büschels mit den Geraden eines Stralenbüschels; diese Kurve hat drei dreifache Punkte, in denen je eine Tangente nach dem Mittelpunkte des Stralenbüschels zielt. Dieser Punkt trägt 15 Wendetangenten der ψ^7; demnach gibt es auf φ^5 15 Punkte, wo φ eine Kurve ε^4 *oskuliert.*

In diesen 15 Punkten wird φ^5 von φ^{14} berührt; weil φ^{14} vierfache Punkte besitzt in den 8 Basispunkten von (k_1) und (k_2), treffen φ^5 und φ^{14} sich noch in 8 Punkten. Diese bilden offenbar 4 Paare von Punkten in denen φ von einer ε^4 berührt wird.

Demnach wird eine *Ebene* i. A. von *vier* Kurven ε^4 je *zweimal berührt* und von *fünfzehn* Kurven ε^4 *oskuliert.*

7. Aus § 4 ergibt sich dass ein Punkt M der Ebene φ acht Tangenten t von Kurven ε^4 trägt. Die in φ liegenden Tangenten t umhüllen somit eine Kurve der achten Klasse; ihr Geschlecht stimmt mit dem der Kurve φ^5 überein, ist daher 6. Sie besitzt somit 15 *Doppeltangenten*; zu diesen gehören die 6 Geradenpaare der Büschel (k_1), (k_2), denn jede dieser 12 Geraden trägt eine Involution I^2, berührt also zwei ε^4.

Die übrigen 3 Doppeltangenten sind offenbar singuläre Bisekanten s (§ 4). Die Geraden s bilden daher eine *Kongruenz* [2,3].

8. Eine andere Abbildung der Kongruenz E ergibt sich, wenn man ε^4 abbildet auf die Spuren der beide Bisekanten durch M in einer Bildebene ε. Eine ε^4 hat alsdann zwei Bildpunkte, E_1 und E_2.

Die durch M gelegte ε_0^4 wird abgebildet auf die Punkte der Kurve ε_0^3, in welche ε_0^4 aus M projiziert wird. Auf ε_0^3 gibt es 11 *singuläre Bildpunkte*: die Spuren der nach M zielenden singulären Bisekanten.

Die *Bildkurve* λ der ε^4, welche die Gerade l treffen, hat auf ε_0^3 7 *vierfache* und 4 *zweifache* Punkte (§ 4), ist somit eine λ^{12}.

Zwei Kurven λ^{12} haben 8 Paare E_1, E_2 gemein; hieraus ergibt sich wiederum $\mathsf{P}^5\mathsf{B}^2\nu^2 = 8$.

Ein System Φ erzeugt Involutionen I^4 auf den Flächen F^4 und Q, welche zu den singulären Bisekanten gehören; diese I^4 haben 12 bez. 6 Doppelpunkte. Demnach hat die Bildkurve von Φ 7 *zwölffache* und 4 *sechsfache* Punkte, wonach ihr Grad 36 beträgt. Mit einer λ^{12} hat sie 24 Paare E_1, E_2 gemein; hieraus erhellt wiederum $\mathsf{P}^5\mathsf{B}^2\nu\varrho = 24$.

Chemistry. — *Osmosis of ternary liquids. General considerations IX.* By F. A. H. SCHREINEMAKERS.

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The isentonic curves and the membrane.

For an osmotic system, in which the substances X, Y and W diffuse through the membrane, obtains among other things, as we have seen in Gen. VI and VII:

a. Eight D.T.'s (diffusion-types) are possible; we find them in scheme I.

SCHEME I.

	X	Y	W
1.	←	←	←
2. (r)	←	←	→
3. (t)	←	→	←
4. (s)	←	→	→
5. (p)	→	←	←
6. (q)	→	←	→
7. (u)	→	→	←
8.	→	→	→

b. The composition of the liquids determines which of the D.T.'s is incongruent and, therefore, not possible.

c. The nature of the membrane determines which of the seven other D.T.'s will occur.

Now we shall first consider fig. 2 Gen. VI, in which *ab*, *cd* and *ef* represent isotonic curves; we shall call the six fields into which these curves divide the triangle, field *p*, field *q* etc. in accordance with the letters, put into them.

We now take the osmotic equilibrium

$$1 \mid M(n) \mid L \qquad \text{fig. 2. Gen. VI} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which on the left side of the membrane is a liquid, represented in fig. 2 Gen. VI by point 1. From our deductions in Gen. VI we know that for this liquid will obtain, when liquid L is situated within:

field *p* then is the D.T. N°. 5 incongruent.

„ *q* „ „ „ „ N°. 6 „

„ *r* „ „ „ „ N°. 2 „

etc. This has also been indicated in scheme I; the letter *p* behind N°. 5

namely means that the D.T. N°. 5 is incongruent, when the liquid is situated in field p ; etc.

If, therefore, we have a system :

$$1 \mid M(n) \mid L(p) \qquad \text{fig. 2. Gen. VI} \quad . \quad . \quad . \quad . \quad (2)$$

in which $L(p)$ is an arbitrary liquid of field p, then the D.T. N°. 5 is incongruent and consequently not possible.

If we have a system :

$$1 \mid M(n) \mid L(q) \qquad \text{fig. 2. Gen. VI} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which $L(q)$ represents an arbitrary liquid of field q, then the D.T. N°. 6 is incongruent and consequently not possible.

So in each of these and the other systems seven D.T.'s are admissible ; the nature of the membrane now will determine which of the seven admissible D.T.'s will occur in each definite case.

We may now illustrate this a little further by the aid of the isentonic curves, which we have discussed already in Gen. VIII ; in order to do this we take an osmotic system :

$$g \mid M(n) \mid L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which the liquid g has a definite composition and is represented, therefore, by a definite point g of the diagram (figs. 1 and 2).

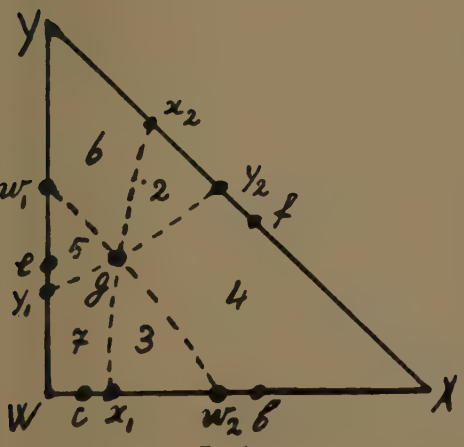

Fig. 1.

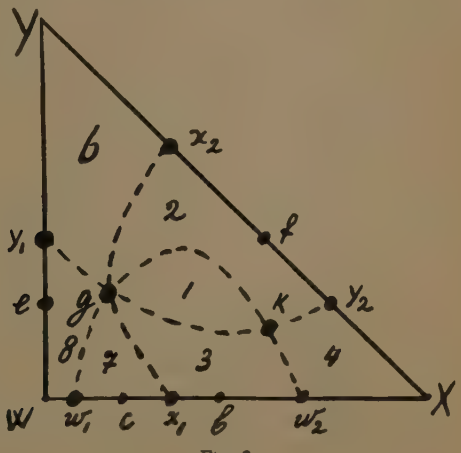

Fig. 2.

Through this point run an isentonic X-curve $x_1\, x_2$, an isentonic Y-curve $y_1\, y_2$ and an isentonic W-curve $w_1\, w_2$. As we have seen in Gen. VIII, these curves may be differently shaped ; for their shape not only depends on the nature of the substances X, Y and W but also on the nature of the membrane.

These isentonic curves divide the triangle into different fields ; in each of those fields we find a figure ; this indicates a D.T. of scheme I, as we shall see below.

We now take a membrane M_1 viz. a membrane, for which the isentonic curves are situated as drawn in fig. 1.

We now imagine liquid L of system (4) somewhere within field 4 ; if we also imagine an isentonic X-curve drawn through this point L, then we see that the liquid g has a greater O.X.n.A. than liquid L ; consequently in system (4) the substance X diffuses towards the left.

If besides we imagine isentonic Y- and W-curves, running through L, then we see that this liquid L has a greater O.Y.n.A. and O.W.n.A. than the liquid g ; so in system (4) the substances Y and W diffuse towards the right. From this appears :

when the liquid L is situated within field 4, then the substances diffuse in system (4) according to :

$$\longleftarrow X \quad \longrightarrow Y \quad \longrightarrow W \quad . \quad . \quad . \quad . \quad . \quad (5)$$

viz. according to the D.T. N^0. 4 of scheme I.

In a corresponding way we are now able to deduce also according to which D.T. the substances will diffuse, when the liquid L is situated in one of the other fields ; this may be found more simply, however, in the following way.

As long as the liquid L is situated in field 4, the D.T. N^0. 4 obtains and consequently the water diffuses towards the right ; when, however, L is situated somewhere on part $g\ w_2$, of the isentonic W-curve $w_1\ w_2$, then no more water passes through the membrane ; when L now passes into field 3 and, therefore, comes on the other side of $g\ w_2$, then the water will diffuse towards the left.

As the directions in which the substances X and Y pass through the membrane, do not change when liquid L passes from field 4 into 3, it follows, therefore, that the diffusion takes place according to :

$$\longleftarrow X \quad \longrightarrow Y \quad \longleftarrow W \quad . \quad . \quad . \quad . \quad . \quad (6)$$

i.e. according to D.T. N^0. 3. Consequently we may say :

when the liquid L is situated within field 3, then the substances in system (4) diffuse according to the D.T. N^0. 3.

When the liquid L passes from field 3 into field 7, therefore, running past part $g\ x_1$ of the isentonic X-curve $x_1\ x_2$, only the direction, in which the substance X passes through the membrane, will change ; so the substances diffuse according to :

$$\longrightarrow X \quad \longrightarrow Y \quad \longleftarrow W \quad . \quad . \quad . \quad . \quad . \quad (7)$$

i.e. according to the D.T. N^0. 7. Consequently we find :

when liquid L is situated within field 7, then the substances diffuse in system (4) according the D.T. N^0. 7.

When the liquid L passes from field 7 into field 5, consequently running past part $g\,y_1$ of the isentonic Y-curve $y_1\,y_2$, then only the direction, in which the substance Y passes through the membrane will change; the substances then diffuse according to:

$$\longrightarrow X \quad \longleftarrow Y \quad \longleftarrow W \quad . \; . \; . \; . \; . \quad (8)$$

i.e. according to the D.T. N^0. 5. So we see:

when the liquid L is situated within field 5 the substances in system (4) diffuse according to the D.T. N^0. 5.

In a corresponding way we find:

when the liquid L is situated within fields 6 or 2, the substances will diffuse in system (4) according to the D.T.'s N^0. 6 or 2.

From this we see that the figure, placed in a field, indicates the D.T. according to which in system (4) the substances diffuse, when the liquid L is situated within that same field.

Now it appears from fig. 1:

in system (4) the substances can diffuse through a membrane M according to all D.T.'s except according to N^0. 1 and N^0. 8; so the three substances cannot at the same time go through the membrane, either towards the left or towards the right.

We now imagine a membrane M_2 viz. a membrane for which the isentonic curves are situated as in fig. 2; herein the curves $w_1\,w_2$ and $y_1\,y_2$ intersect not only in g, but also in a point k. The final points w_1 and w_2 of curve $w_1\,w_2$ are in this figure both situated on the side $W\,X$ of the triangle (compare also fig. 2 Gen. VIII).

In a way corresponding to that for fig. 1 we now find that the figure, placed in a field of fig. 2, indicates the D.T. according to which the substances diffuse in system (4), when the liquid L is situated in that same field.

Now it appears from fig. 2:

in system (4) the substances can diffuse through a membrane M_2 according to all D.T.'s except according to N^0. 5.

Every change in the membrane involves a change in the form of the isentonic curves. In fig. 2 we have assumed among other things that the curves $w_1\,w_2$ and $y_1\,y_2$ intersect not only in g but also in an other point; if we take an other membrane, then the other curves may also have two points of intersection. In fig. 2 we have assumed also that the two final points w_1 and w_2 of curve $w_1\,w_2$ are situated on side WX; when we take an other membrane, however, both these points may also be situated on side WY and it is even possible that curve $w_1\,w_2$ is situated entirely within the triangle and forms, therefore, a closed curve. The same obtains for the curves $x_1\,x_2$ and $y_1\,y_2$.

If, therefore, other membranes are used, diagrams may arise, in which the appearance, the division and the expansion of the fields may differ abso-

lutely from what is seen in figs. 1 and 2 ; it is even possible fields may occur, consisting of two parts, entirely separated from one another.

In further deductions we shall use only the membranes M_1 and M_2 and, therefore, the figs. 1 and 2 ; the reader may apply these considerations also to other cases.

We imagine the figs. 1 and 2 placed upon one another in such a way that the anglepoints W, X and Y coincide ; now we assume that the points g coincide also, so that the liquid g has the same composition in both figures ; of course the isentonic curves and fields of the two figures do not coincide.

Now we take an osmotic system :

$$g \mid M(n) \mid L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

in which L is a liquid, which in fig. 1 is situated within field 4. In fig. 2 this liquid can be situated also within field 4, but also in an other field. If we assume that field 4 of fig. 1 totally or partly covers the fields 1, 2, 3 and 4 of fig. 2, then the liquid L of system (9) may be, therefore, situated within anyone of the fields 1, 2, 3 or 4 of fig. 2. From this follows :

if in system (9) we bring a membrane M_1 viz. a membrane for which the isentonic curves are situated as in fig. 1, then the substances diffuse according to the D.T. N^0. 4, independent of the composition the liquid L has.

if, however, in system (9) we bring a membrane M_2 viz. a membrane, for which the isentonic curves are situated as in fig. 2, then the composition of the liquid L will determine according to which of the D.T.'s N^{os} 1, 2, 3 or 4 the substances will diffuse.

Besides in system (9) with a membrane M_2 transition-D.T.'s may occur, which are not possible with a membrane M_1.

When the liquid L namely is situated on the line dividing fields 1 and 2 and consequently on the isentonic W-curve, no water will diffuse ; then the diffusion takes place according to :

$$\longleftarrow X \quad \longleftarrow Y \quad \longrightarrow W \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which represents the transition between the D.T.'s N^0. 1 and N^0. 2.

When L is situated on the line dividing fields 3 and 4 and consequently also on the isentonic W-curve, the diffusion will take place according to :

$$\longleftarrow X \quad \longrightarrow Y \quad \longrightarrow W \quad . \quad . \quad . \quad . \quad . \quad (11)$$

When L is situated on the line dividing fields 1 and 3 and, therefore, also on the isentonic Y-curve, the diffusion will take place according to :

$$\longleftarrow X \quad \longrightarrow Y \quad \longleftarrow W \quad . \quad . \quad . \quad . \quad . \quad (12)$$

When L is situated on the line dividing fields 2 and 4, the diffusion will take place according to :

$$\longleftarrow X \quad \longrightarrow Y \quad \longrightarrow W \quad . \quad . \quad . \quad . \quad . \quad (13)$$

As a very special case L can be situated also in the point of intersection

K of the isentonic W- and Y-curves ; then the diffusion will take place according to :

$$\longleftarrow X \quad \longrightarrow Y \quad \longrightarrow W \quad . \quad . \quad . \quad . \quad . \quad (14)$$

From this follows, therefore :

If in system (9) we bring a membrane M_1, then the substances X, Y and W always pass through the membrane according to the D.T. N^0. 4 ;

if, however, we take a membrane M_2, then the substances will diffuse according to one of the D.T.'s N^{os} 1, 2, 3 or 4 ; it is also possible, however, that only the substance X or only $X + Y$ or only $X + W$ pass through the membrane. It depends on the composition of the liquid L which of these cases will occur.

It is clear that the transition-types, discussed above, can exist only during a single moment. If namely an osmotic system is left to itself, both liquids will change their compositions ; they run along their osmosispath, so that they are continually represented by other points of the diagram. If for the sake of simplicity we keep the composition of the liquid g constant during the osmosis in some way or other, then the liquid L passes along a path, which terminates in point g. When this path intersects more fields, then the transition-type occurs in the moment that L passes from one field into an other.

We now take the osmotic system

$$g \mid M(n) \mid \textit{water} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

in which pure water is on the right side of the membrane ; so the substances X and Y can only diffuse towards the right side ; the direction in which the water diffuses depends, however, on the nature of the membrane. If in this system (15) we bring a membrane M_1 then point W is situated in field 7 of fig. 1 ; therefore, the substances diffuse according to D.T. N^0. 7 namely :

$$\longrightarrow X \quad \longrightarrow Y \quad \longleftarrow W \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Consequently the water diffuses towards the liquid g.

If, however, we bring a membrane M_2 in this system, then point W is situated in field 8 of fig. 2 ; so the substances diffuse according to D.T. N^0. 8 viz. :

$$\longrightarrow X \quad \longrightarrow Y \quad \longrightarrow W \quad . \quad . \quad . \quad . \quad . \quad (17)$$

We can also imagine a membrane, for which the point w_1 of the isentonic W-curve in fig. 2 coincides with the anglepoint W ; then the diffusion will take place according to the transition-type :

$$\longrightarrow X \quad \longrightarrow Y \quad \longrightarrow W \quad . \quad . \quad . \quad . \quad . \quad (18)$$

so that no water diffuses.

As the substances X and Y diffuse towards the right side, the water in system (15) passes at once into a liquid, which contains all substances.

The curves *ab*, *cd* and *ef*, going through point 1 in fig. 2 Gen. VI are isotonic curves ; we now imagine them drawn also in the figures 1 and 2 of this communication ; we now call them also *ab*, *cd* and *ef* ; in figs. 1 and 2 we only find the points *b*, *c*, *e* and *f* of these curves. The six fields into which these curves divide the triangles are called again, just as before, field *p*, field *q* etc. In order that the diagrams may be more easily compared, we imagine in fig. 2 Gen. VI the point 1 substituted by *g*.

We now imagine the left-side liquid 1 in the osmotic system (2) replaced by *g* ; we then have :

$$\left.\begin{array}{c} g \mid M(n) \mid L(p) \\ 1,\ 2,\ 3,\ 4,\ (5),\ 6,\ 7,\ 8 \end{array}\right\} \quad \ldots\ldots \quad (19)$$

This means : when the right-side liquid is situated in field *p*, the D.T. N⁰. 5 is incongruent and, therefore, not possible ; the seven other D.T.'s are admissible ; the nature of the membrane determines which of these admissible D.T.'s will occur.

We now bring a membrane M_1 in this system. If in fig. 1 we now imagine that part *gb* of curve *ab* has been drawn and part *gf* of curve *ef*, then we see that field *p* (viz. field *gbXf*) is situated within field 4. From this appears : if in system (19) we bring a membrane M_1 then of the seven admissible D.T.'s only the D.T. N⁰. 4 occurs. We represent this by :

$$\left.\begin{array}{c} g \mid M_1 \mid L(p) \quad \text{fig. 1} \\ 4,\ (5). \end{array}\right\} \quad \ldots\ldots \quad (20)$$

If we change the nature of the membrane, the isotonic curves remain unchanged ; their shapes namely only depend on the nature of the substances *X*, *Y* and *W* ; the isentonic curves, however, do change. We can imagine that in fig. 1 point w_2 comes between *b* and *X*, so that field *p* also covers a part of field 3 ; when y_2 comes between *f* and *X*, then field *p* also covers a part of field 2.

If we now take a membrane M_2 so that the isentonic curves are situated as in fig. 2, then field *p* covers the fields 1, 2, 3 and 4 entirely or partly. We then find :

$$\left.\begin{array}{c} g \mid M_2 \mid L(p) \quad \text{fig. 2} \\ 1,\ 2,\ 3,\ 4,\ (5). \end{array}\right\} \quad \ldots\ldots \quad (21)$$

This means: if in system (19) we bring a membrane M_2, then four of the seven admissible D.T.'s can occur viz. N⁰ˢ 1, 2, 3 and 4.

Here it appears, however, that it depends also on the composition of the liquid *L* (*p*) which of those four D.T.'s will occur.

We now take the osmotic system :

$$\left.\begin{array}{c} g \mid M(n) \mid L(q) \\ 1,\ 2,\ 3,\ 4,\ 5,\ (6),\ 7,\ 8 \end{array}\right\} \quad \ldots\ldots \quad (22)$$

in which, as we have seen in (3), the D.T. N⁰. 6 is incongruent and, therefore, not possible.

Now we imagine in figs. 1 and 2 that the curves *gb* and *gc* have been drawn ; then we see that field *q* (viz. the field *bgc*) entirely or partly covers the fields 3, 4 and 7 in fig. 1 and the fields 3 and 7 in fig. 2.

If in system (22) we now bring a membrane M_1 or M_2, we find :

$$\left.\begin{array}{llll} g \mid M_1 \mid L(q) & \text{fig. 1} & g \mid M_2 \mid L(q) & \text{fig. 2} \\ 3,\ 4,\ (6),\ 7 & & 3,\ (6),\ 7 & \end{array}\right\} . \quad (23)$$

Consequently the substances can diffuse through both membranes according to the D.T.'s N⁰. 3 and 7 and through the membrane M_1 according to N⁰. 4 besides.

We now take the osmotic system :

$$\left.\begin{array}{c} g \mid M(n) \mid L(r) \\ 1,\ (2),\ 3,\ 4,\ 5,\ 6,\ 7,\ 8 \end{array}\right\} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

in which the D.T. N⁰. 2 is incongruent and, therefore, not possible. We now imagine that in figs. 1 and 2 the curves *gc* and *ge* have been drawn ; then we see that field *r* (viz. the field *gcWe*) partly covers the fields 5 and 7 in fig. 1 and the fields 7 and 8 in fig. 2. If we now bring a membrane M_1 or M_2 in system (24) it follows, therefore :

$$\left.\begin{array}{llll} g \mid M_1 \mid L(r) & \text{fig. 1} & g \mid M_2 \mid L(r) & \text{fig. 2} \\ (2),\ 5,\ 7 & & (2),\ 7,\ 8 & \end{array}\right\} . \quad . \quad (25)$$

Consequently the substances can diffuse through both membranes according to the D.T. N⁰. 7, besides through M_1 according to N⁰. 5 and through M_2 also according to N⁰. 8.

We now take the osmotic system :

$$\left.\begin{array}{ccc} & g \mid L(bg) & \\ O.X.A. & O.Y.A. & O.W.A. \\ \longleftarrow & \longrightarrow & \text{———} \end{array}\right\} . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

in which the right-side liquid is a liquid of the part *bg* of the isotonic *W*-curve *ab* ; so the two liquids of system (26) have the same O.W.A., as in (26) has indeed been indicated by the dash. If we imagine a point on *bg* (e.g. in fig. 2 Gen. VI) we see, as has already been indicated by the arrows in (26), that this liquid has a smaller O.X.A. but a greater O.Y.A. than the liquid *g*.

Therefore, the substance *X* diffuses through a membrane *M* (*X*) towards the left ; the substance *Y* through a membrane *M* (*Y*) towards the right ; the water, however, does not diffuse through a membrane *M* (*W*).

If we now bring a membrane M_1 in system (26) then *bg* is situated in field 4 of fig. 1 ; if, however, we take a membrane M_2 then *bg* is situated in field 3 of fig. 2 ; consequently we find :

$$\left.\begin{array}{llll} g \mid M_1 \mid L(bg) & \text{fig. 1} & g \mid M_2 \mid L(bg) & \text{fig. 2} \\ \longleftarrow\ \longrightarrow & \longrightarrow 0 & \longleftarrow\ \longrightarrow & \longleftarrow 0 \end{array}\right\} . \quad (27)$$

in which the letters X, Y and W have been omitted ; the sign o indicates that the water diffuses incongruently. From this appears, therefore :

the substance X diffuses through the membranes $M\ (X)$, M_1 and M_2 in the same direction, viz. towards the left ;

the substance Y diffuses through the membranes $M\ (Y)$, M_1 and M_2 in the same direction, viz. towards the right ;

the water does not diffuse through a membrane $M\ (W)$; through the membrane M_1 it goes towards the left and through a membrane M_2 towards the right.

If we take the osmotic system :

$$\left.\begin{array}{ccc} & g \mid L\,(eg) & \\ O.X.A. & O.Y.A. & O.W.A. \\ \longrightarrow & \longrightarrow\!\!\!\!\!\!\!- & \longleftarrow \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

in which on the right side of the membrane is a liquid of part eg of the isotonic Y-curve ef, then both liquids have the same O.Y.A. We see that the arrows point to that side of the membrane where the O.X.A. and the O.Y.A. are greatest.

If we now bring a membrane M_1 in this system (28), then eg is situated in field 5 of fig. 1 ; if, however, we take a membrane M_2 then eg is situated in field 8 of fig. 2 ; so we find :

$$\left.\begin{array}{ccccccc} g \mid M_1 \mid L\,(eg) & & \text{fig. 1} & & g \mid M_2 \mid L\,(eg) & & \text{fig. 2} \\ \longrightarrow & \longleftarrow 0 & \longleftarrow & & \longrightarrow & \longrightarrow 0 & \longrightarrow 0 \end{array}\right\} \quad (29)$$

From this appears :

the substance X diffuses through the membranes $M\ (X)$, M_1 and M_2 in the same direction, namely towards the right ;

the substance Y does not diffuse through a membrane $M\ (Y)$; Y goes through the membrane M_1 towards the left and through a membrane M_2 towards the right ;

the water diffuses through the membranes $M\ (W)$ and M_1 towards the left ; through the membrane M_2, however, towards the right, therefore incongruently.

In the osmotic system :

$$\left.\begin{array}{ccc} & g \mid L\,(fg) & \\ O.X.A. & O.Y.A. & O.W.A. \\ \longleftarrow & \longrightarrow\!\!\!\!\!\!\!- & \longrightarrow \end{array}\right\} \quad . \quad . \quad . \quad . \quad (30)$$

on the right side of the membrane is a liquid of the part fg of the isotonic Y-curve ef. If we bring a membrane M_1 in this system, then fg is situated in field 4 of fig. 1 ; if, however, we take a membrane M_2 then fg intersects

the fields 1 and 2, so that the D.T.'s Nos 1 and 2 and their transition-type can occur ; so we have :

$$\begin{array}{lll} g \mid M_1 \mid L(fg) & \text{fig. 1} & g \mid M_2 \mid L(fg) \quad \text{fig. 2} \\ \longleftarrow \quad \longrightarrow 0 & \longrightarrow & \longleftarrow \quad \longleftarrow 0 \quad \longleftarrow 0 \\ & & \longleftarrow \quad \longleftarrow 0 \quad \longrightarrow 0 \\ & & \longleftarrow \quad \longleftarrow 0 \quad \longrightarrow \end{array} \qquad (31)$$

From this appears :

the substance X diffuses through the membranes $M(X)$, M_1 and M_2 towards the left ;

the substance Y does not diffuse through a membrane $M(Y)$; through a membrane M_1 the substance Y goes towards the right and through a membrane M_2 towards the left ;

the water diffuses through the membranes $M(W)$ and M_1 towards the right ; through a membrane M_2, however, it may diffuse as well towards the right as towards the left and besides it may not diffuse at all ; it depends on the composition of the right-side liquid which of these three cases will occur.

It appears from the cases, discussed above, and the great number of others still to be deduced by the reader, that the composition of the liquids and the nature of the membrane determine the directions, in which the substances diffuse.

(To be continued.)

Leiden, Lab. of Inorg. Chemistry.

Chemistry. — *The osmotic vapour-pressure*, I. By F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of March 23, 1929).

In the osmotic system

$$(L)\ P \mid (water)\ P \qquad \longleftarrow W \quad . \quad . \quad . \quad . \quad . \quad (1)$$

we find on the left side of the membrane a liquid L and on the right side pure water; on both sides the pressure is P (we imagine e. g. $P =$ 1 atmosphere). When this membrane is permeable for water only, the water will diffuse in the direction of the arrow, viz. towards the left.

We now increase the pressure of the liquid L till no more W can diffuse, keeping the temperature constant; then (1) passes into the osmotic equilibrium:

$$(L)\ P_1 \mid (water)\ P \qquad \text{——} W \quad . \quad . \quad . \quad . \quad (1^a)$$

The difference in pressure $P_1 - P$ is the $O.P.$ (Osmotic Pressure); in order to distinguish this from another $O.P.$, which we shall discuss later on, we shall call $P_1 - P$ the increase of $O.P.$

With this increase of pressure the liquid L can remain stable; it can also become metastable, however, or even labil. If e. g. it deposits the solid substance F, then instead of (1^a) arises the osmotic equilibrium

$$(L_1 + F)\ P_2 \mid (water)\ P \qquad \text{——} W \quad . \quad . \quad . \quad (1^b)$$

in which of course L_1 has an other composition than L; consequently the pressure P_2 and, therefore, also the increase of $O.P.$ differs in (1^b) from that in (1^a).

We can also imagine that L when pressure is increased should de-mix into two liquids; we then get:

$$(L' + L'')\ P_3 \mid (water)\ P \qquad \text{——} W \quad . \quad . \quad . \quad (1^c)$$

with an other increase of $O.P.$ than in (1^a) and (1^b).

We can also prevent the diffusion of the water in (1) by lowering the pressure on the right side of the membrane; we then get:

$$(L)\ P \mid (water\ l.v.)\ \pi \qquad \text{——} W \quad . \quad . \quad . \quad . \quad (1^d)$$

When the pressure π is greater than the vapour-tension of the water, this will remain fluid; if, however, π is smaller, we get water-vapour. So it depends on the pressure π whether the water on the right side of the membrane will be present either as a liquid or as vapour; this has been indicated in (1^d) by the letters l and v. Now we shall call the difference in pressure $P - \pi$ the decrease of $O.P.$

If we now compare the decrease of $O.P.$ with the increase of $O.P.$ in system (1), it appears:

the $O.P.$-decrease depends only on the properties the liquid L has under the pressure P;

the $O.P.$-increase, however, depends on the properties this liquid gets, when pressure is increased; in this case it may or may not deposit a solid substance or de-mix into two liquids, etc.

In the osmotic system

$$(L+F)\,P \mid (water)\,P \qquad \longleftarrow W \quad . \quad . \quad . \quad . \quad (2)$$

is a liquid saturated with solid F on the left side of the membrane. We now increase the pressure on the left side of the membrane till an osmotic equilibrium is reached. When the solubility of F increases with increase of pressure, we may distinguish two cases.

a. From (2) arises the osmotic equilibrium

$$(L_1+F)\,P_1 \mid (water)\,P \qquad \longleftarrow W \quad . \quad . \quad . \quad (2^a)$$

in which L_1 has another composition than L.

b. The solid substance F disappears entirely with the increase of P; then we get the osmotic equilibrium:

$$(L_2)\,P_2 \mid (water)\,P \qquad \longleftarrow W \quad . \quad . \quad . \quad . \quad (2^b)$$

in which L_2 has an other composition than L or L_1. This system (2^b) will occur when too little solid F is present in (2) or when by the increase of P the melting-point of F is lowered even below the temperature of the system.

We now imagine in (2) n quant. of L and m quant. of F; the composition of L_1 and the $O.P.$-increase P_1-P then are independent of $n:m$ and, therefore, completely determined. The composition of L_2, however, does depend of $n:m$; so the $O.P.$-increase P_2-P will change with $n:m$ and consequently can have an infinite number of values.

We now lower the pressure on the right side of the membrane in (2) till no more W diffuses; we then get the osmotic equilibrium:

$$(L+F)\,P \mid (water\ l.v.)\,\pi \qquad \longleftarrow W \quad . \quad . \quad . \quad (2^c)$$

with the $O.P.$-decrease $P-\pi$. The system $L+F$ is still present here unchanged, whereas in (2^a) and (2^b) it has passed into an other. From this it appears:

the $O.P.$-decrease only depends on the properties the phases L and F have under the pressure P; this $O.P.$-decrease is independent of $n:m$ and completely determined.

the $O.P.$-increase, however, depends on the phases, formed under increase of pressure; this $O.P.$-increase is dependent on $n:m$ and can have an infinite number of values.

Corresponding considerations obtain also when we put some arbitrary

system or other e.g. $L + F + F'$ or $L + L'$, etc. on the left side of the membrane. So, if we wish to deduce a property of a liquid or of a system with the aid of the $O.P.$, the $O.P.$-decrease is more useful in this respect than the $O.P.$-increase, which not only changes the system, but can also have an infinite number of values. For this reason we shall in our next considerations use the $O.P.$-decrease or a pressure, connected with it.

At first we take an equilibrium

$$(L + W \text{ vapour})\, P_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which there is no membrane between the liquid and the W-vapour. If we keep the temperature and the composition of the liquid constant, this system will exist only under a definite pressure P_0; this is the $W.V.P.$ (Water-Vapour-Pressure) of the liquid. We now imagine this system under a piston; if we slightly increase the pressure on this piston, so that it becomes a little higher than P_0, some W-vapour will condense; if we slightly decrease this pressure, some water evaporates.

We have something like this in the osmotic equilibrium:

$$(L)\, P \;\Big|\; \begin{matrix} a & & d \\ \hline W \text{ vapour} & | & \pi \\ \hline b & & c \end{matrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which $a\,b$ is a membrane, permeable for water only. On its left side is a liquid under the pressure P and on its right side a space $a\,b\,c\,d$ in which W-vapour under the pressure π.

We now imagine that cd is a movable piston; if we slightly increase the pressure on this piston, W-vapour will pass through the membrane towards the liquid; we may say that W-vapour condenses in L through the membrane. If we decrease the pressure on the piston a little, water will evaporate through the membrane. So we have a "condensation" and "evaporation" of water through a membrane; for this reason we shall call π: the vapourtension of the liquid through the membrane or the $O.W.V.P.$ (Osmotic-Water-Vapour-Pressure) of the liquid.

In system (3) liquid and vapour can only exist under a definite pressure P_0, in (4), however, the liquid can exist under an infinite number of pressures P and with every pressure P goes a definite pressure π of the vapour. So we may say:

the $W.V.P.$ of a liquid has a definite value P_0; the $O.W.V.P.$, however, has an infinite number of values π; we shall see that $\pi > P_0$.

We now take the osmotic equilibria

$$\left.\begin{matrix} (L)\ P \mid (W \text{ vapour})\, \pi \\ (L')\, P \mid (W \text{ vapour})\, \pi' \end{matrix}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the first the liquid L has an $O.W.V.P. = \pi$ and in the second the liquid L' an $O.W.V.P. = \pi'$. When L and L' are different, π and π' will generally be different too; when, however, L and L' have the same $O.W.A.$, then $\pi = \pi'$. In order to prove the last statement we form the new osmotic system:

$$\left.\begin{array}{l}(L)\ P \mid (W\ vapour)\,\pi \\ \hline (L')\,P \mid (W\ vapour)\,\pi'\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which also the horizontal dash represents a membrane $M(W)$. Now we imagine each of the two vapour-phases in a closed space. If $\pi > \pi'$, then W-vapour would diffuse from the higher towards the lower vapour-space; as the pressure in the higher space will consequently decrease, water will evaporate from L towards the right; as, however, the pressure increases in the lower vapour-space, W-vapour will condense in L'. As, therefore, the $O.W.A.$ of L increases and that of L' decreases, water will now also diffuse from L' towards L.

If we now regulate the surface of the membranes in such a way that the same quantity of water runs through all membranes in the same time, then neither the compositions of the liquids nor the pressures π and π' will change; we then get an eternal circular current of water and water-vapour.

When $\pi < \pi'$, this circular current would run in the opposite direction.

As we assume that these circular currents are not possible, it follows $\pi = \pi'$; so we find:

all liquids with the same $O.W.A.$ have also the same $O.W.V.P.$ (consequently they have also the same $O.P.$-decrease; their $O.P.$-increase will generally be different).

As all liquids of the isotonic W-curve $a\,b$ (fig. 1 Gen. VI) have the same $O.W.A.$, they consequently will have the same $O.W.V.P.$ too; the same obtains for the liquids of the curve $a_1 b_1$ and for those of $a_2 b_2$. As we shall see further on, however, the liquids of $a_1 b_1$ have a greater $O.W.V.P.$ than those of $a\,b$ and these again a greater $O.W.V.P.$ than those of $a_2 b_2$.

Now we consider the osmotic equilibrium:

$$(L)\ P \mid (W\ vapour)\,\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

We represent the composition of the liquid by

$$x\ mol\ X + y\ mol\ Y + (1 - x - y)\ mol\ W \quad . \quad . \quad . \quad . \quad (8)$$

we shall call the thermodynamical potential of one quantity of liquid and vapour ζ and Z.

Now we imagine that dn quantities of vapour condense through the membrane in the liquid; we now find (comp. e. g. Gen. VI) that (7) is an osmotic equilibrium, when

$$\left(\zeta - x\frac{\partial \zeta}{\partial x} - y\,\frac{\partial \zeta}{\partial y}\right)_p = Z_\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

has been satisfied. The $O.W.A.$ of a liquid viz. ξ_w is, as we have seen in Gen. VI, determined by:

$$\xi_w = -\zeta + x\frac{\partial\zeta}{\partial x} + y\frac{\partial\zeta}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Consequently we may write instead of (9):

$$-(\xi_w)_P = Z_\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

So the $O.W.V.P.$ of the liquid, namely π is determined by its $O.W.A.$ From this follows, what we have already found above in an other way: all liquids with the same $O.W.A.$ also have the same $O.W.V.P.$ From (11) follows:

$$-d(\xi_w) = V\,d\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

this means: the $O.W.V.P.$ of a liquid decreases (increases) when its $O.W.A.$ increases (decreases).

This is indeed clear; the water of the liquid in (7) namely tries to diffuse through the membrane towards the outside; it is prevented to do this however, by the $O.W.A.$ of the liquid itself and by the pressure π of the vapour. So the greater one influence is, the smaller may be the other.

From the above follows among other things:

all liquids of an isotonic W-curve have the same $O.W.V.P.$; this will be greater, the closer this curve is situated to point W (fig. 1 Gen. VI).

Previously we have seen among other things:

the water diffuses through a membrane $M(W)$ towards that side, where the $O.W.A.$ is greatest.

Therefore, we may also say now:

the water diffuses through a membrane $M(W)$ towards that side where the $O.W.V.P.$ is smallest.

If we keep the composition of the liquid in (7) constant, and change the pressure, it follows from (9) or (11):

$$d\pi = \frac{\triangle v}{V}\,dP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

in which, as follows from (9) or (11):

$$\triangle v = v - x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

We see from this that $\triangle v\,.\,dn$ is the increase of the volume of the liquid, when this absorbs dn quantities of water; this increase of volume is generally positive. V is the volume of the W-vapour under the pressure π. From (13) now follows:

with increase of pressure the $O.W.V.P.$ of a liquid increases.

In general, however, $d\pi$ is small with respect to dP; the volume V

of a grammolecule W-vapour under a low pressure π is namely ten thousand times larger than the volume v of a grammolecule water and, therefore, also larger than $\triangle v$. From this appears:

In general the $O.W.V.P.$ of a liquid will increase only a little when the pressure P is raised.

So the osmotic equilibrium (7) is comparatively insensible to a small change in the pressure of the liquid, but very sensible to a small change in the pressure of the vapour.

As a special case of (7) the pressure can also be the same on both sides of the membrane. In system (3) namely the liquid under a pressure P_0 is in equilibrium with vapour under the same pressure P_0; of course this still remains so when we put a membrane $M(W)$ between liquid and vapour. We then get:

$$(L)\, P_0 \mid (W \text{ vapour})\, P_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

in which P_0 is the $W.V.P.$ of the liquid. Therefore, the $O.W.V.P.$ of the liquid is now equal to its $W.V.P.$ In connection with what we have seen above, it appears from this, that the $O.W.V.P.$ of a liquid under a not too high pressure P, will be only very little larger than its $W.V.P.$

If the gas-laws obtain for the vapour, we have $\pi V = RT$; then we may write for (13):

$$RT\frac{d\pi}{\pi} = \triangle v \,.\, dP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Of course the change in volume $\triangle v$ is dependent on the pressure P; between wide limits of P, however, we may look upon $\triangle v$ as constant. Then follows from (16):

$$RT \log \pi = P \,.\, \triangle v + C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

in which C is a constant. As, however, (17) also obtains for system (15) in which $P = P_0$ and $\pi = P_0$, we consequently also have

$$RT \log P_0 = P_0 \triangle v + C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

so that C has been determined. We now find:

$$\log \pi = \log P_0 + (P - P_0)\frac{\triangle v}{RT} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

So if in a diagram we draw $\log \pi$ and P as variables, we will get a straight line.

We now take the two osmotic equilibria:

$$(L)\, P \mid (W \text{ vapour})\, \pi \quad ; \quad (L')\, P' \mid (W \text{ vapour})\, \pi' \quad . \quad . \quad . \quad (20)$$

For them obtain the equations:

$$-(\xi_w)_P = Z_\pi \qquad -(\xi')_{P'} = Z_{\pi'} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

As $Z_{\pi'} - Z_{\pi} = RT \log \frac{\pi'}{\pi}$ it follows:

$$(\xi_w)_P - (\xi'_w)_{P'} = RT \log \frac{\pi'}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

consequently a relation between the $O.W.A.$ and the $O.W.V.P.$ of two different liquids (L and L') under two different pressures (P and P').

If we take e.g. the osmotic system

$$(L)\, P \mid (L')\, P' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

then the first part of (22) represents the difference of the $O.W.A.$ of the two liquids; therefore, the first part will determine the direction in which in (23) the water will diffuse through a membrane $M(W)$; therefore, this direction is determined also by the second part of (22), which contains the $O.W.V.P.$ of the two liquids. Instead of:

no water diffuses through a membrane $M(W)$ when the two liquids have the same $O.W.A.$;

the water diffuses towards that side of a membrane $M(W)$ where the $O.W.A.$ is greatest;

we may also say now, therefore:

no water diffuses through a membrane $M(W)$ when the two liquids have the same $O.W.V.P.$

the water diffuses towards that side of a membrane $M(W)$ where the $O.W.V.P.$ is smallest.

We now imagine the second system in (20) replaced by system (15). Then in (22) we have to put $P' = P_0$ and $\pi' = P_0$. When both liquids now have the same compositions, we can replace the first part of (22) by $(P_0 - P) \triangle v$. We then get:

$$(P_0 - P) \triangle v = RT \log \frac{P_0}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

which is in accordance with (19).

In order to give an other form to (9) we put:

$$\zeta = \varphi + RT\,[x \log x + y \log y + (1 - x - y) \log (1 - x - y)] \,. \quad (25)$$

If we now calculate $\partial\zeta : \partial x$ and $\partial\zeta : \partial y$, we see that we may write for (9):

$$\left(\varphi - x \frac{\partial\varphi}{\partial x} - y \frac{\partial\varphi}{\partial y}\right)_P + RT \log (1 - x - y) = Z_{\pi} \quad . \quad . \quad (26)$$

When the quantities x and y of the substances X and Y become nought, the osmotic equilibrium (7) will pass into:

$$(water)\, P \mid (W\ vapour)\, P_s \,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

in which on the left side of the membrane is pure water. Consequently

P_s is the *O.W.V.P.* of pure water under the pressure P. For this system obtains, as follows from (26):

$$(\varphi_0)_P = Z_{P_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The index o indicates that x and y are nought; therefore, φ_0 is the thermodynamical potential of pure water. From (26) and (28) now follows:

$$\left(\varphi - x\frac{\partial\varphi}{\partial x} - y\frac{\partial\varphi}{\partial y} - \varphi_0\right)_P + RT\,log\,(1-x-y) = Z_\pi - Z_{P_s}. \quad (29)$$

If we now take a much diluted liquid, so that x and y are very small, the term with the index P will only consist of magnitudes of the second and higher orders, which may be neglected; now (29) passes into:

$$RT\,log\,(1-x-y) = RT\,log\,\frac{\pi}{P_s} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

From this follows:

$$P_s - \pi = (x+y)\,P_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

in which the first part represents the decrease of the *O.W.V.P.*; from this appears:

the *O.W.V.P.* of the water (viz. P_s) is lowered by the addition of other substances; at a first approximation this decrease is proportionate to the added quantities $x+y$ of the new substances.

With the aid of the *O. P.*-increase we can deduce several properties; of course we may also do this by the help of the *O.W.V.P.* or the *O. P.*-decrease, consequent upon this. We shall not do so, however; we namely can find all these properties easily and accurately with the aid of the thermodynamical potential [1]), so that a deduction with the aid of the *O. P.*-increase or -decrease themselves, following first from thermodynamical considerations, is an unnecessary round about way. We shall use this *O.W.V.P.* however, which is bound up with the *O.W.A.*, when in subsequent communications we shall consider the diffusion through membranes more in detail.

(To be continued).

Leiden. *Lab. of Inorg. Chemistry.*

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[1]) For the first thermodynamical deduction of the exact formula of the O.P.-increase. its meaning (diffusion-counter-pressure), etc. comp.

J. J. VAN LAAR:

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Zoology. — *On Laevicolica and Dextricolica in Tunicates.* By J. W. VAN WIJHE (Anatomical Laboratory, Groningen).

(Communicated at the meeting of March 23, 1929).

Last year, I suggested that the position of the intestinal loop and anus, morphologically either to the right or to the left side of the animal, would form a fundamental difference between the group of the Copelata (Appendicularians) and the other Tunicates (Acopa).

As terms based on positive characters I proposed *Dextricolica* (= Copelata) and *Laevicolica* (= Acopa [1]).

The morphological median plane (i. e. the plane dividing the thyroid gland and the central nervous system into right and left halves) is better recognizable in the young animal (larva or bud) than in the adult. In the young animal it is generally a flat plane, but in the adult it may have got curves by the shifting of organs.

I trusted there would exist only apparent [2]) but no real exceptions to the rule that in the Dextricolica the loop morphologically lies on the right side of the animal, in the Laevicolica on the contrary on the left side.

Sooner than I could expect this was confirmed *in part* by such a distinguished morphologist and specialist in Tunicates as prof. GARSTANG, who had partly arrived at the same conclusion independently. He says (1928, p. 180) in his very suggestive essay, rich in facts and hypotheses: "In his latest paper (1928, p. 997) VAN WIJHE has anticipated me in drawing attention to the contrast between Appendicularians and other Tunicates in regard to the twist of the intestinal loop, but by overlooking the exceptional cases of *Doliolum* and *Anchinia* he has been unfortunately led into an untenable hypothesis [3]) .... The existence of *Doliolum*, however, with its median gut and anus .... prohibits the sharp division of Tunicates into 'Dextricolica' and 'Laevicolica' ".

So the rule, that in my opinion is applicable to the whole group of

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[1]) The term "*Acopa*" is defective, being based on a negative character (absence of a tail in the adult).

[2]) As an *apparent* exception I mentioned that in Corellidae the intestinal loop in the adult lies on the right side of the gill-basket. But in the young larva DE SELYS LONGCHAMPS (1900) has found that the anus opens in the left peribranchial pouch.

[3]) Namely the hypothesis (GARSTANG, l.c.) "that the right intestine of Appendicularians and the left intestine of other Tunicates together represent the second pair of gill-slits in Amphioxus, of which the left can be regarded as more potent than the right!"

I hope to show in the sequel that the hypothesis is not "untenable".

Tunicates, would become a rule of minor importance if the exception of the order of Doliolids would hold true as GARSTANG maintains.

The point in question is, whether in the young larva of Doliolum the intestinal loop lies either on the right or on the left side of the central nervous system.

Now GARSTANG was not able to solve the question. He says (l. c. p. 100, 101). "Unfortunately the tail of *Doliolum* is not functional as a locomotive organ, and its nerve-cord appears to atrophy at a very early stage .... but a number of striking peculiarities in the tail and other organs corroborate the view of a close relationship with Appendicularians". He enumerates four of them (l. c. p. 101) to none of which I can attribute much force.

The second of these pecularities might have been conclusive, as it concerns the relations between the nerve-cord and the intestine, but it runs: „The nerve-cord, as it passes backwards between the peri-branchial involutions, bends down the *right* side of the future oesophageal [1]) region as in *Oikopleura* (NEUMANN, Taf. II [XII] figs. 21, 22)".

This fact is rather suggestive of Doliolum belonging to the Laevicolica instead of to the Dextricolica (Appendicularians) and we shall see this proved in the bud of Anchinia, where the nerve-cord in its dextral flexure is much more developed than in the larva of Doliolum. In the figures of NEUMANN the nerve-cord in this flexure is very rudimentary as it shows only one cell in the transverse sections.

GARSTANG in his textfigure 5c „hypothetical larva of primitive Doliolid" (l. c. p. 97) shows the nerve-cord passing the "rectum" [2]) at the left side, thus making it to represent one of the Dextricolica like the Appendicularians. This figure includes a *petitio principii* as it intends to demonstrate the argument he was not able to establish. The figure must be wrong, as we shall see in a moment.

We may now pass to Anchinia where *to the observant reader* the case is clear, provided that the description [3]) of BARROIS (1885) be compared with his figures.

In clinging to the text, without paying much attention to the figures,

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[1]) If we compare the larva before cutting (NEUMANN, 1905, Taf. XI, fig. 5) with more advanced stages (Taf. XIII, figs. 1, 2) I believe that the sections (Taf. XII, figs. 21, 22) as they pass by the hinder part of the external cloacal slit, will not show the *oesophageal* but the *stomachal* region of the intestinal primordium. This difference, however, does not influence the considerations in the text.

[2]) The term "rectum" is generally used to denote the whole distal limb of the intestinal loop (the proximal limb being formed by oesophagus and stomach). In various Tunicates the distal limb is microscopically and macroscopically differentiated in two different parts. In these cases it is desirable to restrict the term "rectum" to the part at the anus. The other part, beginning at the stomach, may be termed "colon".

[3]) This description, founded on pretty plenty material, is a revision and considerable extension of the short paper, founded on rather scanty material, by KOWALEVSKY and BARROIS (1883).

one might get the impression that the bud of Anchinia would pass by an Appendicularian-stage. BARROIS was so much under this impression by the striking similarities he had discovered, that he tried in any way to see the distal limb of the intestinal loop in his bud on the right side of the body. His preparations, however, clearly show that its situation is on the morphologically left side.

At the time that the anus acquires its external opening he remarks (l. c. p. 233) "Il existe donc un stade dans lequel l'anus débouche par le fait à *la surface de la peau* (fig. 20 [1]) entre les ouvertures des deux poches cloacales [peribranchial pouches] et à droite du tube nerveux".

The figure here quoted is, however, not fit to demonstrate the position of the anus at the right side of the neural cord. We shall see below that, according to other figures, almost the whole of the distal limb ("rectum") of the intestinal loop remains at the left side of the cord and that only its extreme end with the anus crosses the *dorsal* side of the cord to open a little to the right of the *topographical* median plane.

Speaking on the distal limb of the intestinal loop he says on the same page: "elle n'occupe pas une position rigoureusement médiane mais se trouve légèrement infléchie vers la droite (fig. 23 A, 23 B)".

But the hind part of the nerve-cord (and *pari passu* this part of the morphologically median plane) is also very markedly inflected to the right in fig. 23 B [2]).

On our plate are copied four figures of BARROIS (succession of the stages see footnote [1]). They show two successive buds [3]) in dorsal aspect (fig. 19 A and 20 C), a frontal section through a bud of the next stage (fig. 21 B), and a more advanced bud, seen from the right side (fig. 24).

Fig. 19 A shows the nerve-cord still straight. Swollen at both ends and thinner in the middle it diplays an astonishing resemblance to the primordium of the nerve-cord of a mammal at the time of the appearance of the first mesodermic somites. Of course this resemblance is only a curiosity.

The external openings of the peribranchial sacs are still separate. The stomach lies not quite symmetrically, but already more in the left than in the right half of the body. The "rectum" is developing as a little outgrowth directed dorsalwards from the stomach (not visible in this figure). It has not yet attained the ectoderm.

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[1]) The succession of the numbers of the figures in BARROIS' work (e.g. 19, 20, 21, 22, 23) indicate at the same time the succession of the stages. Different aspects of the same stage are represented by addition of a type (e.g. 23, 23 A, 23 B).

[2]) In the figure quoted (Pl. XI, fig. 23 B) it is not to be seen whether the anus crosses the dorsal or the ventral side of the cord, but that it must be the dorsal side is obvious in various figures of Pl. X e.g. figs. 20, 24, 25, 26, 27.

[3]) BARROIS describes three different series of buds, two of them remaining sterile, only the third developing genital glands to maturity. As in the first series the neural cord is best developed, I shall confine myself especially to this series.

Fig. 20 C, the next stage, is important for our purpose as it shows the curvature of the hind part of the neural cord to *the right side* of the body, so that now the cavity of the stomach lies almost quite to the left side of the cord. This curvature must have been caused by the development of the "rectum" growing dorsalwards from the stomach. We see here the dextral flexure of the cord better developed than in the larva of Doliolum. In this larva the cord is very rudimentary at the flexure and the swollen hind end is absent. It is also lacking or rudimentary in both the other series of buds of Anchinia.

The external openings of the peri-branchial sacs are coalescing on the dorsal side of the neural cord and here the anus appears (not shown in the figure) at this stage.

Fig. 21 B represents a very interesting frontal section. Both thickenings (the rostral and the caudal, not the thinner middle part) of the neural cord are cut. The stomach lies as clear as possible *to the left* of the caudal thickening. The explanation of the figure only says that it is a "coupe horizontale" but a comparison with fig. 20 C is convincing to see that the left side of the figure corresponds to the left of the animal [1]).

Fig. 24 represents a more advanced bud, seen from the right side. It shows clearly that the intestinal loop lies to the left of the neural cord. The posterior thickening of the cord (three large cells lying on it in the figure) lies to the right of the spot where the stomach is continued in the distal limb of the intestine. BARROIS says (l.c. p. 235) [2]) "La portion postérieure du tube nerveux . . . s'infléchit vers la droite, et le renflement arrondi qui la termine vient se placer sur l'intestin [i.e. *to the right* of the intestine, *not dorsal* to it, the figure showing the right side of the bud] au point où ce dernier s'unit avec l'estomac".

To resume: Anchinia belongs to the Laevicolica, its bud having the intestinal loop at the left side of the morphologically median plane.

The same must obtain in the whole group of Doliolids, as in the larva of Doliolum the neural cord shows a rudiment of the dextral flexure (the primordium of the intestine lying at its left side) so well developed in the first series of buds of Anchinia.

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[1]) BARROIS draws the attention to quite another point in which this section is highly remarkable. It shows the walls of the right and left peribranchial sacs continuous with the wall of the pharynx but the opening does not fall in this section. In another section of the same stage, however, the opening is seen.

BARROIS (l.c. p. 260), probably correctly, compares it to the gill-opening of Appendicularians, but he is not sure whether it may close later on or be the same as a narrower one in a little later stage (l.c. p. 237, fig. 24 A) "qui est certainement la première fente branchiale".

[2]) The peripheral nervous system, already visible in this bud is very simple.

After BARROIS (l.c. p. 239) "Cela conduit à donner un schéma du système nerveux périphérique beaucoup plus simple que celui de notre première mémoire" (KOWALEVSKY et BARROIS, 1883).

Besides the unpaired neural cord there are only two paired peripheral nerves, the one supplying the region of the mouth, the other the region of the cloacal opening.

The exception claimed by GARSTANG for the order of Doliolids[1]) does not exist; they belong to the Laevicolica as well as the allied order of Salps and there is no exception to the rule that in Appendicularians the intestinal loop lies at the right of the morphologically median plane, in all other Tunicates at the left.

On the Position of the Anus in Tunicates.

The loop lying lateral, one still might regard the anus as a median formation. If, however, we consider the position of the anal opening in the group of Tunicates apart, free from every prejudice about this opening in other groups, and ask whether the anus is a median or a lateral organ, the weight of evidence is on the side of a lateral, for:

1. In the Laevicolica the *dorsal* anus lies in all orders except one on the left side; only in Doliolids it is found in the topographically median plane[2]) or shifted a little to the right, but Anchinia proves that morphologically, it belongs to the left side (cf. the figures of BARROIS, quoted p. 274 footnote).

2. In the Dextricolica (Appendicularians) the *ventral* anus lies in all groups[3]) except one on the right side, only in Oikopleurinae in the topographically median plane. But as this group is probably the least primitive of Appendicularians it is not probable that the position of their anus would be primitive, the more as its forward extension between the gill-slits is clearly secondary, as GARSTANG already has pointed out.

It is a pity that our knowledge of the embryology of Appendicularians is limited to that of Oikopleura dioica, in its generative organs the most specialized of Tunicates.

Accordingly we come to the conclusion that the anus in Tunicates belongs to the same side as the intestinal loop.

It is a problem how the dorsal anus of Laevicolica may become the ventral anus of Appendicularians (or *vice versa*) and BARROIS in his admirable memoir has tried to solve the question in the case of Anchinia.

After enumerating three important points (1885, p. 260) in which developmental stages of Anchinia resemble the Appendicularians, he says that the chief point of difference lies in the position of the anus, which in Anchinia is placed "entre le cordon nerveux et l'un [the right one] des tubes cloacaux" and he continues (l.c. p. 261) "cependant .... on

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[1]) Dolchinia mirabilis (KOROTNEFF) is found by NEUMANN (1913) to belong to the genus Doliolum and FEDELE (1923) could identify it with the species Doliolum Chuni, NEUMANN, so that it should be named Doliolum mirabile (KOROTN.) FEDELE 1923.

[2]) If we knew the origin of the neural cord in the *larva* of Anchinia, where it (lying in the dorsal median line) in all probability will be a derivative of the ectoderm, we might say that it is *not possible* to consider the anus as a median dorsal organ.

[3]) E.g. the genera Fritillaria, Appendicularia and Kowalevskia, which may be considered as representatives of different groups.

peut supposer que cette inflexion du côté droit se soit exagérée chez l'Anchinie jusqu'à refouler l'anus du côté dorsal".

On this supposition, however, BARROIS would not get an Appendicularian, for, apart from the fact that the chief mass of the intestinal loop would remain on the left side of the bud, only the extreme anal part bending down at the right side, the gut would cross the neural cord *dorsally*, a phenomenon not seen in any Appendicularian.

The same holds true for Laevicolica in general. Their left-sided anus cannot become an Appendicularian one by crossing *over* the neural cord and running ventralwards at the right side of the animal [1]).

Crossing *under* the nerve-cord is phylogenitically excluded, as the anus must always have retained its external opening.

On the other hand it is likewise excluded that the anus of an Appendicularian by moving dorsalwards at the right side and crossing over the neural cord would become an anus of a Doliolid.

Theoretically there is a possibility of avoiding the difficulty with the neural cord, by supposing that the anus has wandered not along the right but along the left side of the animal, crossing the ventral median line.

There is, however, no Appendicularian with the anus on the left side, as might be expected.

This objection holds as well when one tries with BARROIS to derive the anus of Appendicularians from that of Anchinia (or of Laevicolica in general) as when one derives *vice versa* — as may be the general opinion — the anus of Anchinia from that of Appendicularians.

Comparison with Amphioxus.

On account of the striking affinities in the early development of Ascidians and Amphioxus it is generally granted that Amphioxus is the nearest ally of Tunicates.

GARSTANG (1928, p. 155) says „Hitherto the Ascidian tail has been generally regarded as equivalent to the postanal region of Amphioxus, specialized for larval life by degeneration of the myotomes". [2])

This may be true for the majority of workers on Ascidians, but there is a minority, which, on the example of VAN BENEDEN and JULIN (1886) believe that the musculature of the "tail" of the Ascidian larva is represented, not in the postanal, but in the preanal musculature of Amphioxus. They cannot believe, that in Ascidian larvae the whole of the musculature and the corresponding part of the notochord of the trunk of Amphioxus would have disappeared without leaving a trace. They

[1]) Of course the neural cord must have existed still at the time when the anus was supposed to cross the dorsal median line. If it had disappeared already at that time, one would get no Appendicularian at all.

[2]) In the opinion of GARSTANG only the end of the tail in the region of the larval fin of Amphioxus would be the homologue of the Ascidian tail.

neither believe that the intestine would have remained, only curving forwards, without showing this motion in ontogeny. On the contrary, one sees the intestinal loop developing in the body of the Ascidian whilst the entoderm in the tail still forms a straight row of cells.

By my study on the development of Amphioxus I am induced to adhere to the minority and believe that the preanal myotomes of the young larva of Amphioxus with one gill-slit are represented in the body and the tail of the Ascidian larvae. [1])

The rudimentary myotomes (about ten in number) of the Appendicularian tail must also be represented by as many preanal myotomes in the young larva of Amphioxus. In this cryptometameric stage of the larva it is very difficult to observe the segmentation of the myotomes (see my "Verslag" etc. 1928, p. 632, 633) — no wonder that in Appendicularians the rudimentary segmentation is denied altogether by many distinguished observers.

So the majority (with GARSTANG) and the minority have a very different standpoint and the minority cannot assume, that the Tunicate intestine would be homologous with that of Amphioxus (cf. GARSTANG, p. 180).

They must assume that, phylogenetically, the intestine in the tail of Ascidians has disappeared as does its rudiment in ontogeny, the tail only developing the muscular function, and that another part of the gut developed in the meantime the intestinal loop of the Ascidian.

In my opinion this part of the gut has been a second gill-pouch of which in Amphioxus [2]) only the left antimere (moving to the right side of the body) is preserved in the larva. It is in this larva the first in the later row of functional pouches, but disappears in the period of metamorphosis.

In other words (cf. my paper, 1914, p. 71—74) the ancestor of Tunicates was an animal with two pairs of gill-pouches, an antimere of the latter pair being metamorphosed in the intestinal loop, the other antimere disappearing as only one loop is necessary.

In Laevicolica it was the left antimere that developed in this way and it is not impossible that it was so too in Appendicularians. In the case of the Appendicularians, however, we would have the difficulty to assume that the loop of the left side must have wandered to the right side, crossing the ventral median line as the gill-pouch does in Amphioxus.

We saw, however, above (p. 277) that the probability is against such a wandering.

Since the sharp division of Tunicates in Laevicolica and Dextricolica became clear to me, I prefer the simpler supposition that in Appen-

[1]) The preoral ("premandibular") myotome is lost in Tunicates.

[2]) The first pair in Amphioxus being represented by the mouth and the club-shaped gland. GARSTANG regards this gland as a homologue of the epicardial pouch in Tunicates.

dicularians it has been the right pouch, in the other Tunicates the left one, that developed into the intestinal loop[1]).

Appendix.

As the reader probably will find it difficult to believe that the single gill-slit in the young larva of Amphioxus may perform the function of an anus, an observation may have a place here, which, though not conclusive, at least shows the possibility.

Last summer, at the Zoological Station of Naples, I tried to feed these young larvae with microscopically fine carmine grains, which more advanced larvae (with 13—15 gill-slits at Heligoland) took so eagerly. The young larvae, however, refused them generally.

After having tried in vain to feed larvae of an earlier date, I placed a lot of them on the fifth day after fertilization in a watch-glass with the carmine grains. After an hour I saw that in four larvae the ilio-colonring was full with a lump of the red grains, lying there quiet, without rotation.

Watching them to see how the carmine might leave the body, as the end of the rectum still seemed to be without lumen, I saw one of them taking up in the intestine a second lump of carmine grains, which stopped a little way behind the gill-slit. After remaining quiet there for about half an hour, I saw it moving forwards and being expelled by the gill-slit.

After isolating the four larvae in glasses without carmine and watching them until the evening, I did not see any motion in the ilio-colonring. The next morning the carmine had disappeared from the intestine.

I regret not to have watched these larvae during the night, as afterwards I did not succeed in my feeding experiments, so that only the single observation with the second lump of carmine may demonstrate the possibility of the gill-slit performing the function of an anus.

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[1]) I suggest this explanation in the opinion that the anus and loop are lateral organs in Tunicates.

The majority of authors, however, believing that these organs must be considered as morphologically median, must assume that the anus (ventral as in Amphioxus) and the loop have migrated in *Appendicularians* to the right (only in Oikopleurinae the anus remaining median) in *all other Tunicates* migrating dorsalwards to the left.

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EXPLANATION OF PLATE. FIGURES FROM BARROIS (1885).

a Anus.
cl Poches ou tubes cloacaux.
cm Portion moyenne (comm.) du cloaque.
e Endostyle.
g Cellules disséminées.
i Intestin.
m Bandes musculaires.
n Masse nerveuse.
oe Oesophage.
p Cavité du péricarde.
pd Pédoncule.
ph Sac pharyngien.
s Corpuscules du sang?
st Sac stomacal, estomac.

Fig. 19A. Stade vu de dos, montrant le tube nerveux s'étendant d'un bout à l'autre du corps. Grossissem. 310 diam.
Fig. 20C. Face dorsale. Grossissem. 330 diam.
Fig. 21B. Coupe horizontale. Grossissem. 430 diam.
Fig. 24. Stade plus avancé. Grossissem. 260 diam.

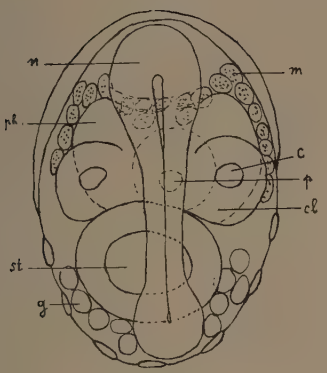

Fig. 19 A.

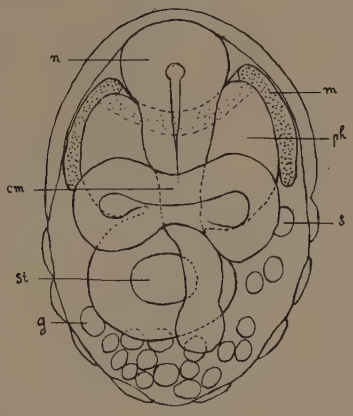

Fig. 20 C.

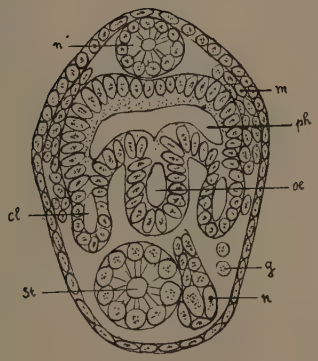

Fig. 21 B.

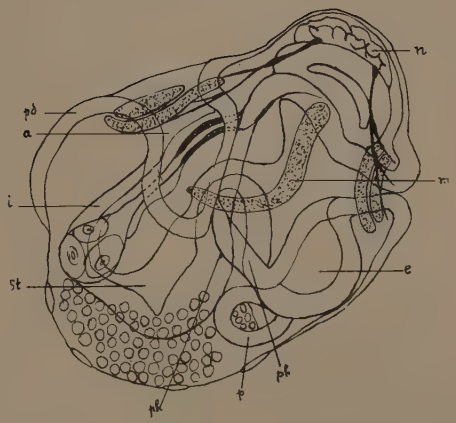

Fig. 24.

Botany. — *Die Funktion des Koffeins im Stoffwechsel von Ilex paraguariensis St. Hil.* Von TH. WEEVERS. (Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of March 23, 1929).

Beim Studium der physiologischen Bedeutung des Koffeins und des Theobromins [1]), das ich im botanischen Garten zu Buitenzorg machte fehlte mir leider die Gelegenheit, auch Ilex paraguariensis, die Matepflanze zu untersuchen. Durch eine Subvention des „Langerhuizen Fonds" instand gesetzt, im Jahre 1925 eine Studienreise nach Brasilien zu machen, konnte ich diese Lücke glücklicherweise ausfüllen. Vorläufige Studien machte ich im botanischen Garten von Rio de Janeiro, aber die eigentlichen physiologischen Versuche wurden in Curityba, dem Mittelpunkte der brasilianischen Matekultur im Staat Parana angestellt.

Im Jahre 1843 isolierten LLOYD BULLOCK und ebenfalls, STENHOUSE [2]) das Koffein aus der Handelsware, PECKOLT [3]) ist dann der erste, der die Blätter einiger Ilex-arten auf ihren Koffeingehalt prüft.

Später jedoch wurde das Vorkommen des Koffeins im Mate mehrfach angezweifelt; so kommt im Jahre 1904 MOREAU DE TOURS [4]) zu der Schlussfolgerung, dass in den Ilex-arten sich ein andres Alkaloid, von ihm Matein genannt, vorfinde. Matein sollte die Elementarformel $C_8H_{11}N_3O_4$ haben und vom Koffein ($C_8H_{11}N_4O_2$) völlig verschieden sein.

Es ist das Verdienst C. C. J. LOHMANNS [5]), gezeigt zu haben, dass die Ergebnisse von MOREAU DE TOURS völlig unzuverlässig sind und lediglich die Folge einer Analyse unreiner Produkte. Die Arbeit LOHMANNS, eine in der portugiesischen Sprache geschriebene Habilitationsschrift, zum Antritt eines Professorats an der Polytechnischen Schule zu Rio de Janeiro, ist jedoch in der botanischen Literatur völlig unbekannt, sodass ich die Hauptsachen hier anführen muss.

Nach Feststellung der Tatsache, das die Matepflanze Koffein und kein Matein enthält, vergleicht LOHMANN den Wert der für die Teepflanze ausgearbeiteten Koffeinbestimmungsmethoden in bezug auf die Mate-

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[1]) TH. WEEVERS. Die physiologische Bedeutung des Koffeins und des Theobromins. Ann. du Jardin botanique de Buitenzorg 2e Ser. Vol. 7.

[2]) STENHOUSE. Liebigs Annalen 1843.

[3]) PECKOLT. Zeitschr. Oester. Apotheker Verein 1882.

[4]) MOREAU DE TOURS. Le maté Etude historique, chimique et physiologique. Paris, 1904.

[5]) C. C. J. LOHMANN. Cafeina ou Mateina. Estudo esperimental sobre al alkaloide principal do mate. Trabalho apresentado a Congregacao da Escola Polytechnica do Rio de Janeiro 1904.

blätter. LOHMANN bekam mit den Methoden von KATZ [1]) und LENDRICH [2]) niedrigere Werte als mit denjenigen von VAN ROMBURGH und LOHMANN [3]) einerseits, von NANNINGA [4]) andererseits. Für erstere Methoden waren die Werte 1.48 und 1.51 %, für letztere 1.67 und 1.74 % und weil das mit NANNINGAS Methode erhaltene Produkt reiner war und der Ertrag grösser, zieht LOHMANN diese Methode den andern vor.

Im Soxhlettapparat wird das angefeuchtete Pulver einige Stunden mit Chloroform extrahiert, dann wird der Chloroform abdestilliert und der Rückstand mit kochendem Wasser aufgenommen.

Die wässerige Lösung wird mit einigen Tropfen basischen Bleiazetats versetzt, filtriert, und im Filtrat mittelst Na_2HPO_4 Lösung der Bleiazetatübermasz entfernt.

Das Filtrat schüttelt man im Scheidetrichter wiederholt mit Chloroform aus, bis alles Koffein aus der wässerigen Lösung weggenommen ist. Nach abdestillieren des Chloroforms wird der schön weisse und kristallinische Rest durch Wägung bestimmt. Wie Kjeldahl Bestimmungen ergaben, enthielt dieser Rest 98 bis 100 % reines Koffein.

Die Arbeit LOHMANNS zeigte, dass der Koffeingehalt, auf Trockengewicht berechnet, war:

in sehr jungen Blättern 2.30 %, in halb erwachsenen 1.50 %, in völlig erwachsenen 0.67 %. Obschon der Gehalt pro Hundert also abnimmt, steigt die absolute Quantität des Koffeins pro 100 Blätter.

100 sehr junge Blätter enthalten ± 44 mg, halb erwachsene + 88 mg, völlig erwachsene + 432 mg.

Die Folgerung LOHMANNS, dass das Koffein ein Dissimilationsprodukt sei, das im Stoffwechsel nicht weiter benutzt werde, ist jedoch nicht berechtigt, denn das Studium der Frage, ob in alten Blättern Koffeinabnahme zu konstatieren sei, hat er völlig vernachlässigt. Dazu wären Bestimmungen der absoluten Quantität pro 100 Blätter unumgänglich gewesen und LOHMANN hat hier blosz prozentische Werte bestimmt.

Er untersuchte ebenfalls den Koffeingehalt der Reiser und Zweige und konstatierte beim Altern eine prozentische Abnahme.

Nach seinen Angaben soll das Koffein ebenso gut im Holze als in der Rinde vorhanden sein, ein Ergebnis, das meinen Beobachtungen bei Coffea, Theobroma, Kola und Paullinia völlig widerspräche.

Im Gegensatz zu den Studien obengenannter Autoren war es meine Absicht, die Funktion des Koffeins im Stoffwechsel der Matepflanze zu studieren. Es schien mir der Mühe wert, diese Frage zu behandeln, weil hier im Gegensatz zu den andern koffeinenthaltenden Pflanzen ein Objekt vorliegt, dessen Blätter zwei bis drei Vegetationsperioden überdauern.

In Südbrasilien und Paraguay (24—26° S. Br.) bleiben die Blätter der

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[1]) KATZ. Siehe LOHMANN l.c.

[2]) LENDRICH. Siehe LOHMANN l.c.

[3]) v. ROMBURGH en LOHMANN. Verslag 's Lands Plantentuin 1896.

[4]) A. W. NANNINGA. Med. 's Lands Plantentuin 1901.

Matepflanze 2—3 Jahre grün und lebendig, sie werden in den dort ziemlich kalten Wintermonaten nicht abgeworfen, während Coffea arabica in San Paulo (23° S. Br.) im Winter kahl steht. In Nord-Italien trifft letzteres auch für Thea sinensis zu, im Gegensatz zu den Tropen, wo die jungen Coffea- und Thea Schösslinge austreiben, bevor die alten Blätter abgefallen sind. Wenn wir in Betracht ziehen, dass es in erster Linie die Blätter sind, welche das Koffein enthalten, so liegt die Möglichkeit vor, dass obige Tatsache einen bedeutenden Einfluss auf den Stoffwechsel der Xanthinderivate ausübe.

Sowie bei meinen früheren Arbeiten war es auch bei der Matepflanze meine Absicht das Vorkommen des Koffeins mikrochemisch mit Hülfe der Methode BEHRENS [1]) zu studieren. Im Gegensatz zu den andern Objekten, hat diese Methode jedoch bei der Matepflanze den Nachteil, dass sich beim Sublimieren ein ölartiger, nicht auskristallisierender Beschlag bildet. beim Sublimieren oft ein ölartiger, nicht auskristallisierender Beschlag bildet. Besonders was dies der Fall bei Untersuchung des in Brasilien gesammelten paraguariensis Exemplars des Amsterdamer botanischen Gartens stets einen gut kristallisierenden Beschlag. Vielleicht liegen hier Varietätsunterschiede vor; nach den anatomischen Bestimmungstabellen LENDNERS [2]) gehört alles Material zu Ilex paraguariensis St. Hil.

Mittels der obenbeschriebenen Methode NANNINGAS war ebensogut in dem brasilianischen Material Koffein nachzuweisen.

Zuerst musste nun untersucht werden, ob wirklich das Koffein, wie LOHMANN behauptete, im Holze zu finden wäre. ± 15—20 mm dicke Aeste wurden sehr sorgfältig bis auf das Kambium abgeschält, die Oberfläche mit einem mit Alkohol angefeuchteten Wattebäuschen abgerieben, dann Holz und Rinde getrennt analysiert.

Trockengewicht Rinde	80 g.	Koffein	588 mg	0.73 %
„ Holz	240 g.	„	Spuren [3])	< 0.005 %

Das Holz enthält kein Koffein, das Kambium wohl; dass LOHMANN zum entgegengesetzten Ergebnis kam, ist dadurch zu erklären, dass die Trennung des koffeinreichen Kambiums vom Holze nicht vollständig durchgeführt wurde.

In dieser Hinsicht d.h. im Fehlen der Xanthinderivate im Holze stimmt die Matepflanze also völlig mit Coffea, Thea, Theobroma, Kola und Paullinia überein.

Früchte und Samen von Ilex paraguariensis enthalten ebenfalls kein oder nur Spuren Koffein, wenigstens weniger als 0.007 %.

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[1]) H. BEHRENS. Anleitung zur mikrochemischen Analyse. Die frischen Blätter werden mit ungelöschtem Kalk zerrieben, mit 96% Alkohol ausgezogen und der alkoholische Extrakt nach Einengen sublimiert.

[2]) A. LENDNER. Mitteilungen aus dem Gebiete der Lebensmitteluntersuchungen und Hygiene veröff. v. schweizer. Gesundheitsamt 1911.

[3]) Höchstens 20 g. ist zugleich zu analysieren, die Ausbeute war dann kleiner als 1 mg.

Die Frage, ob die Koffeinquantität zunimmt, abnimmt oder konstant bleibt, nachdem das Mateblatt erwachsen ist, hatte LOHMANN nicht endgültig gelöst. Zwar konstatierte er, dass der Gehalt pro Hundert auf Trockengewicht berechnet in alten Blättern niedriger ist, aber damit ist die Frage nicht entschieden. Besonders weil aus den Versuchsprotokollen nicht hervorgeht, ob die alten und jungen Blätter von demselben Exemplar herrührten, können individuelle Unterschiede sein Ergebnis beeinflusst haben.

Als ich im Frühjahr (September) in Curityba war, konnte dieser Versuch leicht angestellt werden, indem ich einerseits Blätter der grünen Schösslinge, andererseits Blätter der mit Kork bedeckten Reiser sammelte.

Alle diese Blätter waren völlig erwachsen, erstere jedoch ein Jahr, letztere wenigstens 2 Jahre alt. Von jedem Reis wählte ich ebensoviele ein- als zweijährige gleichgrosse Blätter.

100 1 jährige Blätter Trockengewicht 52 g Koffein 0.98 % = 510 mg
100 2 „ „ „ 54 g „ 0.41 % = 221 mg

Beim Altern nimmt also das Koffein relativ und absolut ab, in den alten, noch grünen und gut lebendigen Blättern war blosz 45 % übrig geblieben.

Diese Tatsache lässt sich auf verschiedene Weisen erklären.

1⁰. Das Koffein wird aus den erwachsenen Blättern nach der Rinde transportiert und dort angehäuft.

In diesem Falle würde beim Altern die Koffeinquantität in der Rinde gewaltig stark ansteigen müssen, denn das Blattvolum übertrifft bei weitem das der Rinde. Schon aus den Arbeiten LOHMANNS kann man folgern, dass diese Erklärungsweise hier nicht zutrifft. Dieser Autor gibt für den Rindegehalt grüner Schösslinge 1.1 % an, für den Gehalt eben mit Kork bedeckter Rinde 0.85 %. Obschon die Rinde letzterer Teile etwas dicker sein wird, ist eine starke Zunahme der Totalquantität in letzterer völlig unmöglich zu nennen. Später nimmt der Koffeingehalt vor wie nach ab, in der Rinde 6 mm dicker Aestchen fand ich 1.25 %, in der 8—11 mm dicker Aeste 0.7 %.

2⁰. Das Koffein wird aus den alten Blättern nach den jungen Schösslingen, insbesondere nach den wachsenden Blättern transportiert.

Diese Voraussetzung ist bei der Matepflanze, wo die jungen Schösslinge sich an der Spitze blatttragender Reiser entwickeln, schwerlich experimentell zu prüfen. Man würde, weil das Holz nie Koffein enthält, auf den Gedanken kommen können die Frage durch Ringelungsversuche zu lösen, aber Ringelung ist bei diesen dünnen Zweigen nicht durchzuführen, überdies hatte zur Zeit meines Aufenthalts in Curityba das Austreiben noch nicht angefangen.

Jedoch kann wider diese Erklärung das bei den andern Koffeinpflanzen beobachtete angeführt werden. Denn dort, wo die Blätter abfallen, bevor die neuen austreiben, nimmt ebensogut die Koffeinquantität beim Altern der Blätter ab.

Die später zu erwähnenden Versuche mit abgeschnittenen und mit den Stielen in Wasser gestellten Blättern sprechen ebenfalls wider diese Annahme.

3⁰. Das Koffein könnte flüchtige N. haltige Verbindungen, speziell Ammoniak abspalten, die durch die Stomata der Blätter ausgeatmet würden.

Nach dem Erscheinen der Arbeit KLEINS [1]) ist die Möglichkeit dieser Erklärung nicht sofort abweisbar.

Bei den später zu erwähnenden Versuchen stellte es sich jedoch heraus, dass von einer Ammoniakabgabe durch Ilex paraguariensis-Blätter nicht die Rede war, wenigstens die abgegebene Quantität so klein war, dass sie vernachlässigt werden konnte. Diese dritte Annahme muss also ebenfalls abgelehnt werden.

4⁰. Das Koffein wird in andere stickstoffhaltige Produkte, vielleicht Ammoniak umgebildet und diese treten aufs Neue in den Stoffwechsel.

Blosz diese Möglichkeit bleibt m. E. übrig und hat abgesehen von allem andern den Vorzug, dass sie sich den bei Thea, Coffea und Paullinia [2]) beobachteten Tatsachen völlig anschliesst.

Die Frage liegt dann auf der Hand, ob ebenfalls hier bei Ilex an eine Umbildung des Koffeins in Baustoffe des Eiweiszes zu denken wäre.

Zur Prüfung dieser Hypothese wurden einige Versuche mit abgepflückten, mit den Stielen in Wasser gestellten Blättern vorgenommen. Stets wurde die Kontrollhälfte beim Anfang der Versuche an den Mittelnerven entlang abgeschnitten und die anderen Hälften mit dem Blattstiel in Wasser gestellt. Am Ende der Versuche wurden diese Hälften ebenfalls vom Mittelnerven abgeschnitten und analysiert. Kontrollversuche zeigten, dass beim Benutzen von 50—100 Blättern, sowohl das Trockengewicht als die Koffeinquantität der Hälften gut vergleichbare Gröszen waren. Bei einer kleineren Anzahl Blätter bestimmte ich mit einem Planimeter die Blattoberfläche und verglich die Koffeinquantität pro 100 cm^2.

Im trocknen Klima von Curityba kamen bei den Versuchen Schwierigkeiten vor; die in Wasser gestellten Blätter trockneten bald aus und starben unter nekrobiotischer Schwarzfärbung ab. Besonders bei den Versuchen im Sonnenlicht war dies der Fall, sodass blosz Versuche im Halbschatten durchzuführen waren. Sogar wenn der Raum, wo die Blätter hingestellt waren, feucht gehalten wurde, konnten die Versuche nur ± 100 Stunden fortgesetzt werden.

1er Versuch 50 Blatthälften 4 Tage im Halbschatten.
Trockengewicht 13 g Koffein 129 mg = 0.99 %
50 Kontrollhälften beim Anfang analysiert
Trockengewicht 12.9 g Koffein 157 mg = 1.22 %
Koffeinabnahme 28 mg = 18 % des Totals.

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[1]) G. KLEIN und M. STEINER. Jahrb. f. wiss. Botanik 1928.

[2]) TH. WEEVERS. Concerning the function of caffein in the metabolism of Paullinia cupana. These Proc. Vol. 29, 1926.

2er Versuch 100 Blatthälften 4 Tage im Halbschatten.
Trockengewicht 28.1 g Koffein 170 mg = 0.61 %
100 Kontrollhälften.
Trockengewicht 28.4 g Koffein 199 mg = 0.70 %
Koffeinabnahme 29 mg = 15 % des Totals.

3er Versuch 50 Blatthälften 4 Tage im Dunkeln.
Trockengewicht 12.4 g Koffein 74 mg = 0.60 %
50 Kontrollhälften.
Trockengewicht 14.2 g Koffein 65 mg = 0.46 %
Koffeinzunahme 9 mg = 14 % des Totals.

Falls bei den ins Dunkel gestellten Blättern die Dissimilation überwiegt [1]) so nimmt das Koffein zu, dagegen nimmt bei den Halbschatten-Versuchen, wo Assimilation möglich ist, das Koffein ab.

Bei letzteren Versuchen blieb die Eiweiszquantität fast die gleiche, während doch aus den Arbeiten MOTHES [2]) und RUHLANDS [3]) gefolgert werden muss, dass in jedem lebenden Pflanzengewebe fortwährend Eiweiszdissimilation stattfindet. Letztere muss dann der Eiweiszsynthese die Wage halten und diese Synthese kann auf Kosten des gespaltenen Koffeins stattfinden, es sei denn dass dieses Koffein zuerst in einfachere Stickstoffverbindungen gespalten sei.

Dass bei den im Dunkeln stehenden Blättern das Koffein zunimmt, während es in den beleuchteten abnimmt, beweist auch, dass im letzteren Falle Ammoniakausatmung nicht die Ursache der Abnahme sein kann.

Dennoch schien es mir wünschenswert, die Frage einer eventuellen Ammoniakabgabe durch lebende Ilex paraguariensis Blätter experimentell zu prüfen. Der obere Teil einer jungen, 20 erwachsene Blätter tragenden Matepflanze, wurde in einen gut geschlossenen Glaszylinder gebracht: blosz der untere Teil blieb auszerhalb des Zylinders, sodass die Pflanze bewässert werden konnte. Mit Hülfe einer Wasserstrahlluftpumpe wurde ein langsamer Luftstrom durchgezogen, wobei eine Glaskapillare die Regulierung der Durchströmungsschnelligkeit ermöglichte.

Zuerst passierte dieser Luftstrom eine mit verdünnter Schwefelsäure gefüllte Pettenkofersche Röhre, sodass alle Ammoniakspuren aus der Atmosphäre zurückgehalten wurden, dann passierte er ein mit feuchter Glaswolle gefülltes, auf 25° C. erwärmtes Gefäss, zur Sättigung des Luftstromes mit Wasserdampf.

Nach dem Glaszylinder mit der Matepflanze passierte der Luftstrom zuerst eine Waschflasche mit Wasser, dann eine mit verdünnter Salzsäure

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[1]) Bei einem andern Versuch im Dunkeln stand gegenüber eine Koffeinzunahme von 10 mg. pro 400 cc. Blättoberfläche eine Eiweissabnahme von 69 mg.

[2]) MOTHES. Die Bedeutung der Säureamide für den Stickstoffwechsel der höheren Pflanze. Planta 1926).

[3]) RUHLAND und WETZEL. Die Wechselbeziehungen im Stickstoff- und Säure-stoffwechsel. Planta 1926.

gefüllte Flasche sodass eventuell von der Pflanze ausgeatmetes Ammoniak zurückgehalten wurde.

Nach 7 Tagen, als die Pflanze noch völlig unbeschädigt war, wurde der Versuch beendet, der Inhalt beider Waschflaschen mit Natronlauge schwach alkalisch gemacht und destilliert. Das Destillat [1]) wurde in eine durch 0.04 cc 0.1 N H_2SO_4 hellrot gefärbte Methylrotlösung aufgefangen. Weil kein Farbumschlag dann zu beobachten war, hatte die Pflanze in 7 Tagen weniger als 0.04 cc 0.1 N . NH_3 also < 0.05 mg Ammoniak produziert. Bei einem zweiten Versuch war die produzierte NH_3 Quantität $\pm$ 0.02 mg m.a.W. so klein dass sie bei meinen Versuchen vernachlässigt werden konnte; für ein ganzes Jahr berechnet, ist der Wert von 1 mg NH_3 pro 20 Blätter verschwindend klein in bezug auf die Koffein-abnahme.

Zusammenfassung.

Ilex paraguariensis St. Hil. enthält nicht Matein wie MOREAU DE TOURS behauptet hat, sondern Koffein. Das Xanthinderivat ist in Blatt und Rinde vorhanden, fehlt jedoch im Holz und in der Frucht.

Beim Auswachsen der Blätter nimmt die Koffeinquantität, pro Hundert des Trockengewichts berechnet, fortwährend ab, obschon die absolute Quantität zunimmt. Nachdem das Blatt erwachsen ist, fängt auch eine Abnahme der absoluten Quantität an, sodass im zwei Jahr alten Blatte mehr als die Hälfte des Koffeins verschwunden ist. Dieses Verschwinden lässt sich nicht durch Transport nach der Rinde und ebensowenig durch Abspaltung und Ausatmung Ammoniaks erklären. Wie ich schon früher in bezug auf die andern Koffein, und Theobromin enthaltenden Pflanzen darlegen konnte, ist auch hier die Erklärung, dass das Koffein wieder aufs Neue in den Stoffwechsel tritt.

Bei abgeschnittenen und in Wasser gestellten Mate-blättern nimmt das Koffein bei verdunkelten Blättern zu, bei beleuchteten ab, sodass die Folgerung, das Koffein werde bei den Dissimilationsprozessen gebildet, könne jedoch aufs Neue zu Synthesen benutzt werden, auf der Hand liegt.

[1]) Sowie der Geruch des Destillats zeigte, waren Methylaminen ebenso wenig ausgeatmet worden.

Physics. — *Ueber den Zusammenhang der Einsteinschen einheitlichen Feldtheorie mit der Quantentheorie.* Von I. TAMM (Moskau). (Communicated by Prof. P. EHRENFEST.)

(Communicated at the meeting of March 23, 1929).

In der vorliegenden Note versuche ich zu zeigen, dass fuer die neue Einsteinsche Feldtheorie [1]) gewisse quantenmechanische Zuege charakteristisch sind und dass man hoffen darf, dass diese Theorie die Erfassung der Quantengesetze des Mikrokosmos ermoeglichen wird.

I. Der Hauptgedanke der neuen Einsteinschen Theorie besteht in der Annahme, dass die physikalischen Eigenschaften des Raum-Zeit-Kontinuums durch die Beschaffenheit der lokalen parallelen 4-Beine bestimmt werden. Unter einem 4-Bein ist die Gesamtheit von 4 zu einander orthogonalen Einheitsvektoren zu verstehen; deren auf ein beliebiges Koördinatsystem X^ν bezogene Komponenten mit ${}_sh^\nu$ und die zugehörigen normierten Unterdeterminanten mit ${}^sh_\nu$ bezeichnet werden. Die Beine gleichen Nummers der lokalen 4-Beine in 2 beliebigen Weltpunkten sind definitionsgemäss als parallel zu einander anzusehen (Fernparallelismus).

Es gilt:

$$ {}^sh_\mu \, {}^sh_\nu = g_{\mu\nu} \quad , \quad {}_sh^\mu \, {}_sh^\nu = g^{\mu\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1) $$

wo $g_{\mu\nu}$ der metrische Fundamentaltensor ist. Der einfachste Tensor der aus ${}_sh^\nu$ und ihren Ableitungen gebildet werden kann, ist der Tensor [2])

$$ \Lambda^\lambda_{\mu\nu} = -\Lambda^\lambda_{\nu\mu} = \frac{1}{2} \, {}_sh^\lambda \left(\frac{\partial \, {}^sh_\mu}{\partial x^\nu} - \frac{\partial \, {}^sh_\nu}{\partial x^\mu} \right) \quad . \quad . \quad . \quad . \quad (2) $$

Ist $\Lambda^\lambda_{\mu\nu}$ gleich Null, so ist die Welt (pseudo-) euklidisch. Die Spur des Tensors, $\Lambda^\lambda_{\mu\nu}$ soll nach EINSTEIN dem elektromagnetischen Viererpotentiale Φ_μ gleich sein; wir setzen etwas allgemeiner

$$ \Lambda^\lambda_{\mu\nu} = a \Phi_\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3) $$

die universale Proportionalitätskonstante a soll später bestimmt werden.

2. Die von DIRAC [3]) gegebene Wellengleichung des Elektrons in einem feldfreien Raum lässt sich unter Verwendung der Minkowskischen Massbestimmung folgendermassen schreiben:

$$ F \psi \equiv (A^s p_s + Bmc) \psi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4) $$

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[1]) A. EINSTEIN, Sitzungsber. d. Preuss. Akad., **17**, 1928; **1**, 1929.

[2]) In seiner letzten Abhandlung lässt EINSTEIN den Faktor $1/2$ in (2) fort.

[3]) P. A. M. DIRAC, Roy. Soc. Proc., (A) **117**, 610 und **118**, 351, 1928.

wo A^s (bzw. B) die Komponenten eines konstanten q-Vektors (bzw. einer skalaren q-Zahl) bedeuten, die in Matrixform ausgedrückt werden können und folgenden Relationen genügen:

$$A^q A^s + A^s A^q = 2\,\delta_{qs} \quad , \quad A^q B + B A^q = 0 \quad , \quad B^2 = 1 \; . \quad . \quad (5)$$

Um die Wellengleichung des Elektrons in einem elektromagnetischen Felde zu bestimmen, hat man nach DIRAC in (5) p_s durch $p_s + \frac{e}{c}\,\Phi_s$ zu ersetzen. Wir werden dagegen aus der folgenden Annahme ausgehen:

Bezieht man die Vektor A^s und p_s auf die Einsteinschen parallelen 4-Beine, so behält im Wesentlichen die Wellengleichung auch in einem beliebigen Felde dieselbe einfache Form (4), wie in dem feldfreien Falle; nur wenn der Vektor $\underset{\lambda}{\Lambda_{\mu}}$ von Null verschieden ist, muss in (4) noch ein Zusatzglied[1])

$$i\,\mu\,\psi = i\,K\sqrt{(\underset{\lambda}{\Lambda_{\mu}}\;\underset{\nu}{\Lambda_{\mu}})}\;\psi \quad , \qquad K = \frac{1}{2\pi}\,h \quad . \quad . \quad . \quad . \quad (6)$$

eingeführt werden.

Will man aber die Wellengleichung auf ein beliebiges Koordinatensystem x^{ν} beziehen, so hat man das skalare Produkt A^s_{ps} entsprechend umzuformen; also statt (2)

$$F\,\psi \equiv (A^{\nu}\,p_{\nu} + Bmc + i\mu)\,\psi \quad , \quad A^{\nu} = {}_s h^{\nu}\,A^s \quad . \quad . \quad . \quad (7)$$

zu schreiben. Um den Vergleich der Wellengleichung (7) mit der Schrödingerschen zu ermöglichen gehen wir zu der Gleichung zweiter Ordnung in ψ über, indem wir die linke Seite der Gleichung (7) der zu F konjugierten Operation F^*

$$F^* = A^{\nu}\,p_{\nu} + Bmc - i\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

unterwerfen. Ersetzt man p_{ν} durch $-\,i\,K\,\frac{\partial}{\partial x^{\nu}}$ und beachtet (3) und (5), so erhält man nach einer einfachen Rechnung:

$$\left.\begin{aligned} F^* F\,\psi \equiv -\,K^2 \Big\{ & g^{\mu\nu}\left(\frac{\partial^2}{\partial x^{\mu}\partial x^{\nu}} - \Gamma^{\lambda}_{\mu\nu}\frac{\partial}{\partial x^{\lambda}}\right) + 2\,a\Phi^{\nu}\,\frac{\partial}{\partial x^{\nu}} + \\ & + a^2\Phi^{\mu}\Phi_{\mu} - \frac{m^2c^2}{K^2} - A^{\mu}\,A^{\nu}\,\Lambda^{\lambda}_{\mu\nu}\frac{\partial}{\partial x^{\lambda}} - aA^{\nu}\,\frac{\partial}{\partial x^{\nu}}\sqrt{\Phi^{\mu}\Phi_{\mu}}\Big\}\,\psi = 0 \end{aligned}\right\} \quad (9)$$

wo $\Gamma^{\lambda}_{\mu\nu}$ die aus dem metrischen Tensor $g_{\mu\nu}$ gebildeten Dreiindicessymbole sind. Nimmt man nun an, dass

$$a = \pm\,\frac{ie}{cK} = \pm\,\frac{2\pi ie}{ch} \quad , \quad e > 0 \; . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

ist, so fällt der von dem Matrizenvektor A^{ν} unabhängige Anteil der Gleichung (9) mit der allgemein-relativistischen Schrödingergleichung

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[1]) Das Unterstreichen eines Index soll nach EINSTEIN das Heraufziehen des Index andeuten.

vollkommen zusammen. Das vorletzte Glied von (9) ist nahe verwandt mit dem in der Diracschen Wellengleichung vorkommenden Gliede

$$\frac{e}{c} K A^{\mu} A^{\nu} F_{\mu\nu} \psi,$$

das von den magnetischen Eigenschaften des Elektrons Rechenschaft gibt; das letzte Glied in (9) ist von derselben Grössenordnung.

Ersetzt man in (9) die Wellenfunktion ψ durch $e^{\frac{iS}{K}}$, und geht dann zu der Limite $K = O$ über, so erhält man die klassische Hamilton-Jakobische Differentialgleichung [1])

$$\underset{K=0}{Lim} \frac{1}{\psi} F^{*} F\psi = g^{\mu\nu} \left(\frac{\partial S}{\partial x^{\mu}} \frac{\partial S}{\partial x^{\nu}} \pm \frac{2e}{c} \Phi_{\mu} \frac{\partial S}{\partial x_{\nu}} + \frac{e^2}{c^2} \Phi_{\mu} \Phi_{\nu} \right) + m^2 c^2 = 0 \quad (11)$$

Das Vorzeichen der Ladung e in (11) hängt davon ab, welcher von den beiden komplex-konjugierten Werten von *a* in die Wellengleichung (9) eingesetzt wird.

3. Die Tatsache, dass das skizzierte Verfahren wirklich zu einer vernünftigen Wellengleichung führt, ist deshalb besonders interessant, weil der zu der erwähnten wellenmechanischen Annahme analoge "klassische" Ansatz: die auf die 4-Beine bezogene Bewegung des Elektrons sei immer gleichförmig, zu keinen brauchbaren Bewegungsgleichungen führt. Somit erscheint *in der Einsteinschen Theorie das wellenmechanische Prinzip dem Prinzipe des kürzesten Weges der geometrischen Optik übergeordnet*. Der Uebergang zu der klassischen Mechanik, d. h. zu der Limite $K = \frac{1}{2\pi} h = 0$ kann erst *nach* dem Einsetzen der Wellenfunktion in die Wellengleichung vorgenommen werden, denn der in der das Verhältniss zwischen $A^{\lambda}_{\mu\lambda}$ und Φ_{μ} bestimmenden Konstante *a* enthaltene Faktor *K* kann nur durch den entsprechenden Faktor, der in der Wellengleichung vorkommt, aufgehoben werden.

4. Da Φ_{μ} reell ist, so folgt aus (3) und (10), dass der Fundamentaltensor $_{\mu}h^{\nu}$ komplex sein muss. Diese Tatsache scheint mit einigen Eigentümlichkeiten der Einsteinschen Feldgleichungen eng verbunden zu sein. Indem EINSTEIN die elektromagnetischen Potentiale Φ_{μ} gleich $A^{\lambda}_{\mu\lambda}$ setzt, definiert er nämlich die Tensordichte

$$\mathfrak{B}^{a}_{kl} = \mathfrak{B}^{a}_{kl} - \varepsilon \mid h \mid (\Phi_l \, \delta^{a}_{k} - \Phi_h \, \delta^{a}_{l}) \quad . \quad . \quad . \quad . \quad . \quad (12)$$

---

[1]) Hätten wir das Zusatzglied $i\mu\psi$ in (7) nicht eingeführt, so wäre das Glied $\frac{e^2}{c^2}\Phi^{\mu}\Phi_{\mu}$ in (11) abwesend. Man könnte aber statt dessen auch *m* in (4) durch $m' = \sqrt{m^2 - \frac{1}{c^2} K^2 A^{\lambda}_{\lambda\mu} A^{\nu}_{\mu\nu}}$ ersetzen; welches von den beiden Verfahren das richtige ist, können wir zur Zeit nicht entscheiden.

wo $|h|$ die aus ${}_s h^\nu$ gebildete Determinante ist und ε eine beliebig kleine Hilfsgrösse bedeuten soll; $\mathfrak{B}^a_{kl}$ ist eine bestimmte aus ${}_s h^\nu$ gebildete Tensordichte. Die Einsteinschen Gravitationsgleichungen lauten

$$\overline{\mathfrak{B}}^a_{k\,l}{}_{/l} - \overline{\mathfrak{B}}^s_{k\,t}\, \Lambda^a_{st} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

mit der Vorschrift nach Vornahme der Operation $/_l$ (Divergenzbildung) zu $\varepsilon = 0$ überzugehen.

Nimmt man nun die Gültigkeit der Beziehungen (3) und (10) an, so kann man (12) unter Weglassung [1]) von ε folgendermassen schreiben:

$$\overline{\mathfrak{B}}^a_{kl} = \mathfrak{B}^a_{kl} \pm i \frac{c}{e} K (\Lambda^\tau_{l\sigma}\ \delta^a_k - \Lambda^\tau_{k\sigma}\ \delta^a_l) \quad . \quad . \quad . \quad . \quad . \quad (14)$$

wobei die Vorschrift zu $\varepsilon = 0$ überzugehen durch die Vorschrift $K = \frac{1}{2\pi} h \to 0$ ersetzt ist. In der Limite fällt der mit i multiplizierte Anteil von (14) fort. Ueberhaupt tritt in erster Näherung bei $K = 0$ und bei Abwesenheit der Materie *die Scheidung der Gesetze der Elektrizität und der Gravitation ein*, was von dem hier vertretenen Standpunkte *der Scheidung* der reellen und der imaginären Anteile des Tensors ${}_s h^\nu$ entspricht.

$$({}^s h_\mu\ {}^s h_\nu = g_{\mu\nu} \text{ reell}, \quad \Lambda^\lambda_{\mu\lambda} = a \Phi_\mu \text{ imaginär}).$$

Da die Gleichungen (13) und (14) bei $K = 0$ in erster Näherung mit den klassischen Gravitationsgleichungen übereinstimmen, so möchte man vermuten, dass bei endlichem K die Einsteinschen Feldgleichungen die wesentlichen Quantenzüge des Mikrokosmos richtig wiedergeben werden.

Moskau, 14 März 1929.

Institut der theor. Physik d. I. Staatsuniversität.

[1]) Vielleicht soll ε nicht weggelassen, sondern durch einen endlichen Zahlenfaktor ersetzt werden.

Medicine. — *Etude sur les Phénomènes oscillatoires dans les Perturbations de la Fonction cérébelleuse chez l'Homme et chez le Chien.* Par H. DE JONG. (Travail des cliniques et laboratoires des Prof. B. BROUWER d'Amsterdam, MAGNUS d'Utrecht et GUILLAIN de Paris.) (Communicated by Prof. B. BROUWER.)

(Communicated at the meeting of March 23, 1929).

Dans un article précédent [1]) nous avons montré qu'il existait une confusion dans la terminologie clinique des phénomènes oscillatoires de la sclérose en plaques. Après avoir étudié ces phénomènes à l'aide de méthodes objectives dans 57 bras de malades, nous avons pu mettre en évidence de quoi il s'agissait dans les divers cas et nous avons défendu finalement la conception que le terme „tremblement intentionnel" n'a plus aucune raison d'être.

Jusqu'à présent il existe une même confusion dans les perturbations de la fonction cérébelleuse. On y parle de dysmétrie (dans le sens de dyscoördination), de tremblement cérébelleux, de tremblement intentionnel, etc. sans analyser à fond ces phénomènes et sans faire le diagnostique différentiel entre eux. Il était donc intéressant d'effectuer les mêmes recherches que nous avons réalisées dans la sclérose en plaques, dans le domaine cérébelleux.

Avant d'exposer les faits, receuillis à ce sujet, nous montrerons par quelques exemples tirés de la littérature le manque de clarté dans ce domaine.

Citons d'abord DEJERINE [2]), A la pag. 477 de son livre il écrit :

„Ce tremblement cérébelleux (BABINSKI, ANDRÉ THOMAS ET JUMENTIE, GORDON HOLMES) est en général un tremblement, à grandes oscillations, et ne se produisant qu'à l'occasion des mouvements volontairs ou du maintien d'une attitude ; c'est un tremblement intentionnel se rapprochant beaucoup de celui de la sclérose en plaques et relevant de la dysmétrie."

A la page 412 : (où il y a question de mouvements cérébelleux.)

„Les mouvements manquent de mesure. Chez le singe (d'après les expériences de LUCIANI, de FERRIER et TURNER) le fait est également très net : ces derniers auteurs comparent le tremblement des membres antérieurs, pendant la préhension des objets, au tremblement intentionnel de la sclérose en plaques."

A la page 422 :

„Ils (les mouvements) ne sont pas exécutés en un seul temps ; ils ne sont pas

[1]) Proceedings de l'Académie Royale des Sciences d'Amsterdam, Vol. **32**, N°. 1, 1929, p. 48.

[2]) Sémiologie des Affections du Système nerveux, Paris, 1926.

continus comme le mouvement normal. Ils sont en quelque sorte discontinus et cette discontinuité du mouvement (ANDRÉ THOMAS) ou tremblement intentionnel, est encore un symptome important des lésions cérébelleuses.

Le tremblement n'existe pas au repos : Il se produit dans deux conditions : 1°. l'Exécution d'un mouvement ; 2°. Le maintien d'une attitude. Le tremblement est donc à la fois cinétique et statique. Il est plus marqué au début de l'exécution de l'acte ou du maintien de l'attitude ..... il rappelle à un degré moindre, le tremblement intentionnel de la sclérose en plaques.

Citons maintenant ANDRÉ THOMAS [1]).
A la page 812 de sa Pathologie du Cervelet il écrit :

„Le tremblement a été signalé comme un symptôme habituel des lésions cérébelleuses, cependant cette désignation ne paraît pas constamment appropriée au trouble décrit sous ce nom.

Au lieu d'être continu, le mouvement est interrompu par des secousses de nombre variable, parfois très peu nombreuses, de sorte que si dans les cas les plus accentués l'anomalie du mouvement rappelle le tremblement intentionnel typique de la sclérose en plaques, dans un très grand nombre de cas, elle est caractérisée par le fractionnement ou la discontinuité du mouvement. Peu importe d'ailleurs le terme ; le principal est de s'entendre sur le fait."

Dans le traité de neurologie allemand d'OPPENHEIM [2]), on trouve que la question de savoir, s'il existe un tremblement cérébelleux, n'est pas encore définitivement résolu et qu'il existe un tremblement cérébelleux, lié au tremblement intentionnel, qui, d'après HOLMES et MILLS, dépend du faisceau rubro-spino-cérébelleux.

Il ressort de toutes ces citations qu'il manque de clarté et de précision dans le domaine que nous venons explorer. Jusqu'à présent il existe notamment une confusion entre tremblement et dysmétrie (ataxie).

Dans mon travail précédent j'ai pu mettre en évidence qu'il est très bien et souvent même facilement possible de distinguer le tremblement et la dysmétrie.

Voici les points principaux :

1°. Le tremblement d'action peut se produire sous l'influence de tout stimulant externe ou interne, y compris les mouvements. La dysmétrie cérébelleuse par contre se produit exclusivement pendant le mouvement. Elle est donc *exclusivement locomoteur*. Le tremblement d'action est *entre autres un phénomène d'ordre locomoteur*.

2°. *Le tremblement d'action* est un phénomène *absolument rythmique*, tandis que la dysmétrie cérébelleuse est *non-rythmique* et irrégulière.

3°. Dans la dysmétrie cérébelleuse, le membre oscille comme un tout. Dans le tremblement d'action, des régions musculaires circonscrites peuvent trembler.

4°. Dans les cas de *tremblements d'action*, on peut observer le phénomène de *„décharge postérieure"*, c'est à dire qu'il y a des décharges dans des cellules motrices, se manifestant comme des tremblements dans les muscles, non seulement pendant l'action du stimulant qui provoque les

[1]) Pathologie du Cervelet. Nouveau traité de médecine. **19**, p. 755—981.
[2]) Lehrbuch der Nervenkrankheiten, Berlin 1923.

décharges, mais également quelque temps après que le stimulant n'agit plus. Dans la dysmétrie cérébelleuse il n'y a pas de phénomène postérieure ; les oscillations cessent immédiatement quand le mouvement, ou la contraction musculaire, cessent. *La dysmétrie cérébelleuse est l'effet de mouvements ou de contractions musculaires exécutées d'une manière trés irrégulière.*

La première mode d'objectivation avec laquelle j'ai trouvé cette différence, consiste en un enregistrement myographique sur un kymographe. Une capsule de MAREY est fixée p. ex. au muscle biceps par une bande autour du bras. Une deuxième méthode très simple et applicable cliniquement permet d'obtenir une approximation suffisante à ce sujet.

Voici brièvement la méthode :

I. On demande au malade (auquel on ne permet pas de faire reposer la main sur un appui), de tirer une ligne d'encre ou de crayon, successivement par les deux mains.

II. Le malade pose maintenant la main, dans laquelle la plume se trouve, sur un appui. L'expérimenteur fait bouger le morceau de carton par exemple de haut en bas sur le style „en repos" et il le répète pour les deux mains. S'il existe une dysmétrie, on verra alors une ligne plus ou moins droite. S'il y a un tremblement, avec des décharges postérieures, on voit des oscillations se dessiner.

III. Si l'on doute encore on met le malade en action, et après on exécute la deuxième épreuve.

Pour tous détails, je me réfère à l'article précédent sur les phénomènes oscillatoires dans la sclérose en plaques.

a. Résultats obtenus par la méthode myographique.

La fig. 1 montre le phénomène irrégulier n'existant que pendant les mouvements et les contractions musculaires, différent fondamentalement des phénomènes de la fig. 2, qui montre un vrai tremblement, le tremblement d'action cérébelleux.

Dans ce cas le phénomène strictement rythmique avec des décharges postérieures, etc. — pouvait être provoqué dans la main gauche en demandant au malade d'exécuter des mouvements de l'autre côté [1]).

l'Emotion et d'autre formes d'action peuvent provoquer le même phénomène. M. THOMAS a également décrit un tremblement cérébelleux provoqué d'une manière analogue. Mon interprétation de tels cas est qu'il s'agit toujours d'un même phénomène et que les modes de provocation sont subordonnées à l' "action" en général.

b. Résultats dans une série de cas obtenus par la méthode de tirer des lignes.

Nous avons pratiqué la deuxième méthode sur une série de cas à Amsterdam et à Paris. Les résultats sons exposés dans les tableaux suivants.

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[1]) Voir Nederl. Tijdschr. v. Geneesk. 1927, I, N°. 18 et Revue neurologique, mai 1928.

[2]) Les fig. 1 et 3 se trouvent également en „Proceedings" de l'Académie Royale des sciences Vol. **32** N°. 1, 1929.

TABLEAU N°. 1.

8 Maladies cérébelleuses et 1 cas de maladie de Friedreich du service du Prof. GUILLAIN à la Salpêtrière.

Malade	Salle	Diagnostique	Epreuve I	Description	Epreuve II	Epreuve III	α = début β = fin du mouvement
1. DESJEAN	Prus	Hémisyndrôme cérébelleux par obus	dr. g. 	dr. ligne droite g. incoördin. grave	dr, g. 	dr. g. 	dr. = coté sain g. $\alpha > \beta$
2. BAZIN	Rayer	Syndrôme cérébelleux		dr. et g. incoörd. légère			dr. $\alpha = \beta$ g. $\alpha = \beta$
3, DELANNEAU	„	Syndrôme cérébelleux		„			dr. $\alpha = \beta$ g. $\alpha = \beta$
4. BENOIT	Cruveilhier	Tumeur cérébelleux		„			dr. $\alpha = \beta$ g. $\beta > \alpha$
5. MATHIEU	„	Syndrôme cérébelleux		dr. inc. gr. g. inc. très gr.			dr. $\alpha = \beta$ g. $\beta > \alpha$
6. ELISABETH	Rayer	„		dr. et g. inc. légère g. avec tremblement d'act.		 trembl. d'action	dr. $\alpha = \beta$ g. $\alpha = \beta$
7. BARTHÉLÉMY	„	Atrophie cérébelleuse		dr. et g. inc. légère dr. + tremblement d'action		 trembl. d'action	dr. $\alpha = \beta$ g. $\alpha = \beta$
8. MENUSIER	„	Syndrôme cérébelleux		„			dr, $\beta > \alpha$ g, $\alpha = \beta$
9. PERROT		Maladie de Friedreich		„			dr. $\alpha = \beta$ g. $\alpha = \beta$

17 bras. Le bras droit du premier malade était du côté sain. Dans tous les cas il y avait une dysmétrie de divers degrés. Dans deux bras il y avait la combinaison avec le tremblement d'action cérébelleux.

$\alpha = \beta$.... 13 fois
$\alpha > \beta$.... une fois
$\beta > \alpha$.... 3 fois

} α indique le début, β la fin d'un mouvement. L'opinion de CHARCOT que „le tremblement intentionnel" s'augmente à la fin du mouvement ne se trouve ici qu'en 3 cas ($\beta > \alpha$).

TABLEAU N°, 2.

Maladies cérébelleuses. 3 cas de la clinique neurologique de l'université d'Amsterdam.

1. H.	Tumeur ponto-céréb.	Dysmétrie avec tremblement d'action (du côté malade) céréb.	$\alpha = \beta$	Voir la fig. 2
2. D.	Tumeur ponto-céréb.	Dysmetrie (du côté malade).	$\alpha = \beta$ dr.	„ „ „ 1
3. H.	Atrophie céréb.	Dysmétrie des deux côtés.	$\alpha = \beta$ dr. $\alpha = \beta$ g.	

Enregistrement myographique (4 bras). $\alpha = \beta$ tous les 4 fois.

TABLEAU N⁰. 3.

3 Maladies cérébelleuses de la clinique neurologique et un cas d'abcessus cérébelleux de la clientèle privée. Enregistration de la méthode de tirer des lignes.

1. d. H. atr. céréb.	dr. g.	incoörd. légère dr. et g.	début et fin. dr. $\alpha = \beta$ g. $\alpha = \beta$	Epreuve II
2. J. Atrophie céréb.		„	dr. $\alpha > \beta$ g. $\beta > \alpha$	
3. B. absc. céréb. (g.)		„ (g.)	g. $\alpha = \beta$	
4. R. Tumeur céréb.		incoörd avec tremblement d'action	dr. $\alpha = \beta$ g. $\alpha = \beta$	

$\alpha = \beta$.... 5 fois
$\alpha > \beta$.... 1 fois
$\beta > \alpha$.... 1 fois

Le bras droit du cas N⁰. 3 était homolatéral du côté sain.
Dysmétrie: 5 fois.
Dysmetrie avec tremblement d'action: 2 fois.

Somme toute il ressort des tableaux le résultat suivant:

Tableau	Nombre des cas	Nombre des bras	Dysmétrie	Combinaison de dysmétrie et tremblement d'action	$\alpha = \beta$	$\alpha > \beta$	$\beta > \alpha$
N⁰. 1	9	17	15 fois	2 fois	13 fois	1 fois	3 fois
N⁰. 2	3	4	3 „	1 „	4 „		
N⁰. 3	4	7	5 „	2 „	5 „	1 „	1 „
	16	28	23 „	5 „	22 „	2 „	4 „

Il y a donc, comme dans la sclérose en plaques de la dysmétrie dans la majorité des cas (environ 80 %) et le reste se compose de la combinaison de dysmétrie avec tremblement d'action cérébelleux.

Jamais il n'y avait de „tremblement intentionnel" spécifique.

Dans 4 cas le phénomène oscillatoire était augmenté à la fin du mouvement ($\beta > \alpha$), comme CHARCOT l'a décrit pour son tremblement intentionnel, c.à.d. que ce fait ne se trouve que dans une minorité très limitée des cas.

c. Phénomènes oscillatoires cérébelleux chez le chien.

Nous avons également eu l'occasion d'étudier des phénomènes cérébelleux chez l'animal grâce à la bienveillance et le secours du Pr. RADEMAKER dans le laboratoire de feu de Pr. MAGNUS à Utrecht. Dans un autre travail [1]) j'ai pu mettre en évidence que chez quelques chiens sans cervelet (opérés par M. RADEMAKER) on peut constater deux

[1]) Loc. cit.

phénomènes : 1°. un tremblement d'action cérébelleux avec tous les caractères d'un vrai tremblement et dont la dépendance de l'action pouvait être démontré par le changement des positions de la tête. Une position, susceptible d'augmenter le tonus, selon MAGNUS et DE KLEIJN, provoqua le tremblement qui disparut au moment du changement de position de la tête.

2°. Il existait un phénomène irrégulier que j'avais décrit de la manière suivante : „On a expérimenté avec trois chiens, auquel le cervelet avait été enlevé ; deux de ces chiens chancelaient distinctement de tout le corps en marchant, en étant debout et en plaçant les jambes".

C'est alors la dysmétrie cérébelleuse.

(J'avais constaté antérieurement la ressemblance de ce deuxième phénomène avec le tremblement intentionnel de la sclérose en plaques qui le plus souvent n'est pas autre chose que la dysmétrie cérébelleuse.)

La fig. 3 montre des graphiques, obtenus par enregistrement myographique (capsule musculaire poussé contre un muscle quelconque d'une jambe) sur un kymographe, provenant du même chien.

Comparons maintenant les phénomènes chez l'homme et chez le chien : Dans tous les deux cas il y a la dysmétrie et le tremblement d'action qui ne diffèrent pas essentiellement quant à leur aspect graphique chez l'un et chez l'autre. Mais il y a une différence dans la division quantative des phénomènes. Chez le chien, la dysmétrie et le tremblement d'action cérébelleux sont aussi fréquent l'un que l'autre, tandis que le tremblement d'action cérébelleux chez l'homme est beaucoup moins fréquent que la dysmétrie. Il se pourrait que cette différence portât sur une différence spécifique entre la pathologie cérébelleuse humaine et animale. Mais cette hypothèse n'est guère vraisemblable, les phénomènes eux-même étant tout à fait identique dans les deux cas. Il n'y a pas de différence essentielle, mais c'est *la relation* entre les deux phénomènes qui diffère. C'est pourquoi je suis enclin à croire que la différence réside plutôt dans un autre facteur qui est également d'ordre relatif. Chez les animaux, le cervelet avait été enlevé tout à fait, tandis qu'on trouve chez les malades presque toujours une partie du cervelet intacte. D'autre part on avait pu observer que la malade, qui présentait le tremblement d'action de la fig. 2 avait une très grande tumeur cérébelleuse, trop grande pour être opérée.

On pourrait donc supposer que le tremblement d'action cérébelleux est l'expression d'un *grand* défécit de substance cérébelleuse. Dans les lésions moins vastes il y a seulement de la dysmétrie. Le fait que les phénomènes dits cérébelleux se manifestent quand il y a ablation totale du cervelet (chez le chien) prouve que les symptômes cérébelleuses ont leur origine en dehors du cervelet. L'énergie qui se déplace pendant l'„action" du ne peut pas entrer au cervelet et les cellules motrices de l'écorce cérébrale p. ex. peuvent être surchargés par cette énergie. Il semble q'une petite surcharge donne lieu à la dysmétrie. Comme cela est-il possible ? Tel est le problème qui se pose maintenant. Mais mes expériences à ce sujet ne sont pas encore

suffisamment avancées pour être publiées. Dans le cas où la surcharge des cellules motrices augmente à cause d'un déficit plus grand de substance cérébelleuse, on obtiendra des décharges rythmiquee sous l'influence de l'action, comme je l'ai exposé dans des travaux antérieurs pour toute phénomène rythmique du système nerveux.

Résumé et Conclusions finales.

Jusqu'à présent il n'y avait pas de clarté dans le domaine des phénomènes oscillatoires dans la perturbation de la fonction cérébelleuse, dû au fait qu'on ne distinguait pas à fond la dysmétrie (dyscoördination) et le tremblement d'action. Nous croyons avoir réalisé cette distinction par deux méthodes objectives. Sur 28 bras nous avons constaté 23 fois de la dysmétrie et 5 fois la combinaison de dysmétrie et tremblement d'action. Chez 3 chiens sans cervelet, nous avons trouvé toujours la combinaison de dysmétrie et tremblement d'action cérébelleux. J'ai expliqué le fait, que cette combinaison était plus fréquente chez le chien que chez l'homme, par la remarque que chez les animaux, il y avait ablation totale du cervelet, fait qui prouve que les phénomènes cérébelleux se produisent hors du cervelet : peut-être dans les circonvolutions rolandiques.

Comme dans la sclérose en plaques, il était nulle part question d'un tremblement intentionnel selon la terminologie clinique. Nous croyons donc que le terme „tremblement intentionnel" n'a plus aucune raison d'être, ni dans la sclérose en plaques, ni dans le domaine cérébelleux.

H. DE JONG: Étude sur les Phénomènes oscillatoires dans les Perturbations de la Fonction cérébelleuse chez l'Homme et chez le Chien.

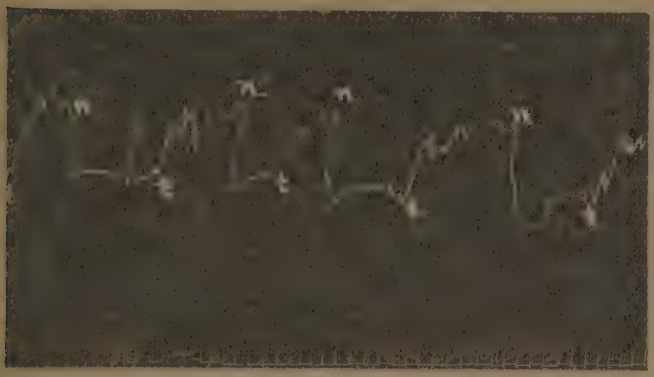

Fig. 1. Malade D. Tumeur ponto-cérébelleuse Entre t. et n. la malade exécute l'épreuve du doit sur le nez. Enregistrement myographique.

Fig. 2. Tremblement d'action cérébelleux dans un cas de tumeur cérébelleuse, contrôlé par l'opération.
Entre les flèches A et B la malade a été mise en action par l'exécution de mouvements de l'autre côté. Après B on voit l'effet des "décharges postérieures".

Fig. 3. Chien sans cervelet.

suffisamment avancées pour être publiées. Dans le cas où la surcharge des cellules motrices augmente à cause d'un déficit plus grand de substance cérébelleuse, on obtiendra des décharges rythmiquee sous l'influence de l'action, comme je l'ai exposé dans des travaux antérieurs pour toute phénomène rythmique du système nerveux.

Résumé et Conclusions finales.

Jusqu'à présent il n'y avait pas de clarté dans le domaine des phénomènes oscillatoires dans la perturbation de la fonction cérébelleuse, dû au fait qu'on ne distinguait pas à fond la dysmétrie (dyscoördination) et le tremblement d'action. Nous croyons avoir réalisé cette distinction par deux méthodes objectives. Sur 28 bras nous avons constaté 23 fois de la dysmétrie et 5 fois la combinaison de dysmétrie et tremblement d'action. Chez 3 chiens sans cervelet, nous avons trouvé toujours la combinaison de dysmétrie et tremblement d'action cérébelleux. J'ai expliqué le fait, que cette combinaison était plus fréquente chez le chien que chez l'homme, par la remarque que chez les animaux, il y avait ablation totale du cervelet, fait qui prouve que les phénomènes cérébelleux se produisent hors du cervelet, peut-être dans les circonvolutions rolandiques.

Comme dans la sclérose en plaques, il était nulle part question d'un tremblement intentionnel selon la terminologie clinique. Nous croyons donc que le terme „tremblement intentionnel" n'a plus aucune raison d'être, ni dans la sclérose en plaques, ni dans le domaine cérébelleux.

H. DE JONG: Étude sur les Phénomènes oscillatoires dans les Perturbations de la Fonction cérébelleuse chez l'Homme et chez le Chien.

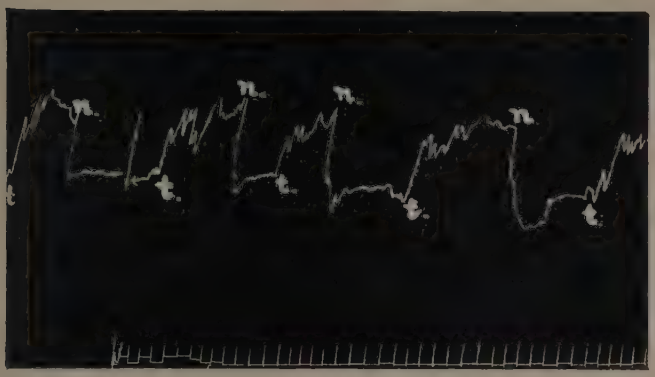

Fig. 1. Malade D. Tumeur ponto-cérébelleuse Entre t. et n. la malade exécute l'épreuve du doit sur le nez. Enregistrement myographique.

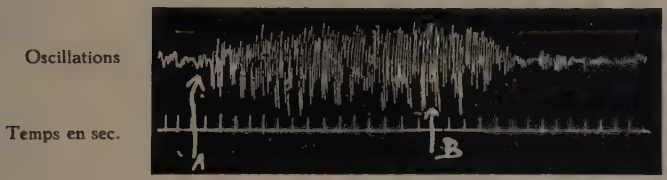

Fig. 2. Tremblement d'action cérébelleux dans un cas de tumeur cérébelleuse, contrôlé par l'opération.
Entre les flèches A et B la malade a été mise en action par l'exécution de mouvements de l'autre côté. Après B on voit l'effet des "décharges postérieures".

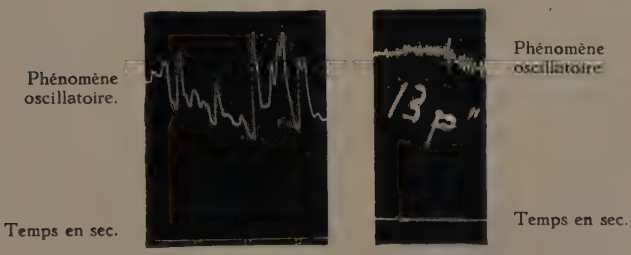

Fig. 3. Chien sans cervelet.

Anatomy. — *The Phylogenetic Development of the Substantia Gelatinosa Rolandi.* Part. II. *Amphibians, Reptiles, and Birds.* By E. KEENAN. M. B., National University of Ireland. (From the Central Dutch Institute for Brain Research, Amsterdam.) (Communicated by Prof. C. U. ARIËNS KAPPERS.)

(Communicated at the meeting of March 23, 1929).

In the first contribution on this subject, published in these Proceedings (Vol. XXXI, 1928), I dealt with the posterior horn regions of fishes. My observations led me to conclude that the substantia gelatinosa Rolandi is general throughout the fishes. The vertebrates below the fishes (Amphioxus and the cyclostomes) do not admit of comparison with higher forms in this respect, owing to the absence of differentiated anterior and posterior horns. Among the fishes it was found that, with the appearance of posterior horns, substantia gelatinosa was recognizable. Lissauer's zone, however, as a separate and recognizable area, is absent, while the body of the horn proper of man is, in part at least, represented in the undivided mass of gray matter, the *corpus commune posterius*, situated behind the central canal.

Among the *amphibians* the following species were examined: — Urodeles: Megalobatrachus maximus, Amblystoma tigrinum, Molge cristata, Proteus anguinus, and Necturus maculatus.

Anures: Pipa pipa, Bombinator pachypus, Bufo vulgaris, Rana esculenta, and Rana catesbyana.

To COGHILL (1909, 1913, 1914, and 1915) and HERRICK (1915), we are indebted for our fuller knowledge of the nervous mechanism in *amphibian* larvae, where the appearance of the cord and its cellular and fascicular arrangements are quite different from those observed in the fully developed animal. The relations in these larvae resemble those of *Amphioxus* and the *cyclostomes* more closely than they do those of *fishes* or adult amphibians, consequently they do not lend themselves for comparison with higher forms in the present instance.

With the passage of the larval stage, and the approach to the adult condition of the animal, the cord undergoes marked changes; and a completely new nervous mechanism, consisting of extra-spinal ganglia, new intra-spinal tracts, and an intra-segmental reflex arc, appears.

The adult amphibian cord as represented by the *bull frog* (Rana catesbyana) is shown in figs. 1, 2 and 3, sections through the cervical swelling, the middle thoracic, and the sacral regions, respectively.

While the cord shows, in many respects, a low development, approaching

the selachians more closely than the teleosts, as has been pointed out by V. LENHOSSÉK (1895), and also has been demonstrated by KAPPERS (1918)

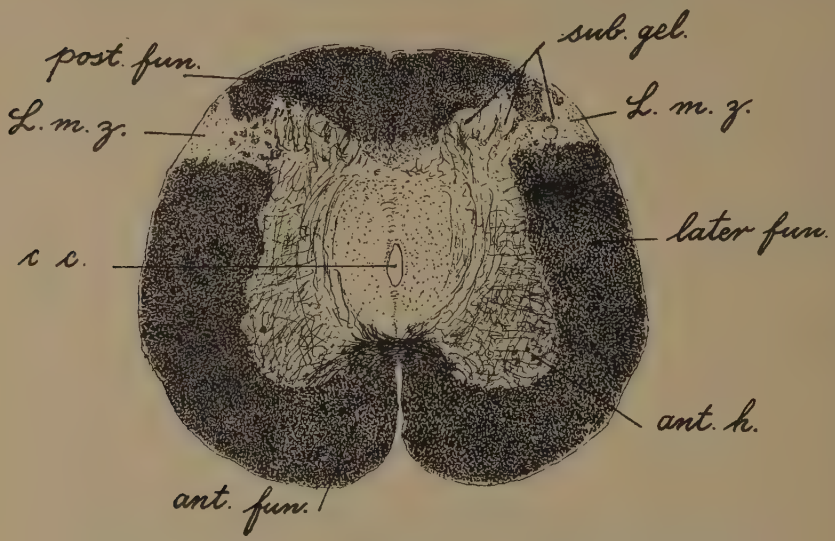

Fig. 1. Transverse section through the cervical enlargement of the spinal cord of Rana catesbyana × 34.

ABBREVIATIONS FOR ALL FIGURES

a = tract of finely myelinated fibres.
b = bundle of coarse fibres.
c = posterior region of the gray matter.
ant. fun. = anterior funiculus.
ant. h. = anterior horn.
ant. r. = anterior root.
ant. r. f. = anterior root fibres.
ant. sept. = anterior septum.
b. of p. h. = body of the posterior horn.
c. c. = central canal.
later. fun. = lateral funiculus.
L. m. z. = Lissauer's marginal zone.
marg. nuc. = marginal nucleus of Gaskell.
med. nuc. = median nucleus of Schwann.
nuc. Burd. = nucleus of Burdach.
post. fun. = posterior funiculus.
post. r. = posterior root.
post. sept. = posterior septum.
sub. gel. = substantia gelatinosa.

for the medulla oblongata and the fore-brain, we find many points indicating the more advanced position of the animal in phylogenetic status. The contrast between Rana catesbyana and the teleosts in the size of the posterior funiculi is very marked. In the former, the accumulation of fibres in the posterior funiculi forces the posterior horns far apart, so that they form with each other a very obtuse angle. BROUWER (1915) estimates that, in the ordinary frog, the proportion of the posterior funiculi to the total white matter in the cervical region of the cord is about 13 %, while in fishes it is only about 5 %. The percentage is much higher in the bull frog. As pointed out by WALLENBERG (1907) for Rana temporaria, this high percentage is, in part, due to descending fibres of the fifth, eighth, ninth,

and tenth cranial nerves, which descend to a very low level in frogs, the trigeminal reaching as far as the lumbar swelling.

Fig. 2. Transverse section through the thoracic region of the spinal cord of Rana catesbyana × 34.

The posterior horn reaches the surface of the cord, and, at its apex, there is an area containing widely-separated finely-medullated fibres, the rudiment of Lissauer's marginal zone of mammals. It is not so clearly defined in amphibians as in reptiles (figs. 4, 5, and 6). The remainder of the posterior horn consists chiefly of substantia gelatinosa, which stretches medially almost as far as the posterior septum, and is broken up into smaller masses by the passage of fibres between the posterior funiculi and the more ventral gray matter of the cord.

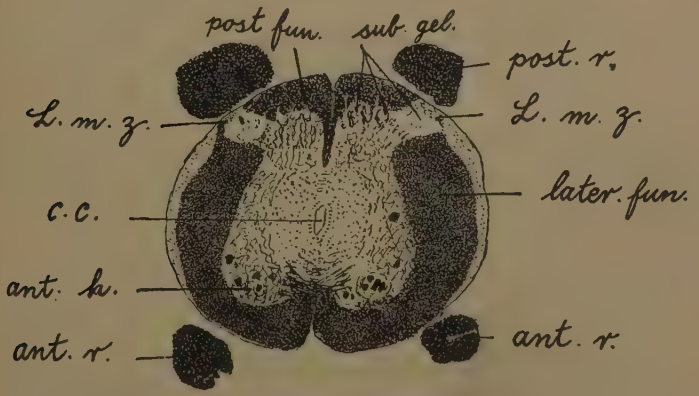

Fig. 3. Transverse section through the lumbo-sacral region of the spinal cord of Rana catesbyana × 34.

The corpus commune posterius of fishes is still represented in the amphibian cord, though it is not such a well-defined and separate body as in sharks, but is continuous with the general mass of gray matter surrounding the central canal, from which the anterior horns project ventro-laterally as short conical protuberances. This relationship exists throughout the entire length of the cord. Cranially, the substantia gelatinosa is continuous with the Rolandic substance of the descending fifth root. It is reduced in amount in the mid-dorsal region, and there is a slight relative increase again towards the caudal end of the cord. It is not so well developed in amphibians as in most members of the teleosts, resembling, in fact, more closely the condition seen in sharks.

The posterior roots cross the apex of the posterior horns in the region designated Lissauer's marginal zone, and pass towards the posterior funiculi. The massing of the root fibres in Lissauer's zone renders it difficult to say, with certainty, whether the roots contribute fibres to this region.

Of *reptiles*, the following species were examined: — Chelonia: Dammonia subtrijuga, and Chelone mydas.

Crocodilia: Crocodilus porosus, and Caiman sclerops.

Lacertilia: Varanus salvator, and Lacerta agilis.

Rhiptoglossa: Chamaeleon vulgaris.

Ophidia: Boa constrictor, and Python reticulatus.

In the *reptilian* cord, the three subdivisions of the posterior horn regions are well marked and easily recognised. The posterior funiculi are strongly developed, and the posterior horns are definitely separated from each other. It is interesting to note, in this connection, that reptiles are the lowest animals in which posterior funicular nuclei have been definitely recognised.

At the upper end of the cervical cord, the relations are as follows: the body of the posterior horn projects backwards and laterally from its junction with the anterior horn; the substantia gelatinosa is massive, and presents a wide border peripherally, where the marginal zone of Lissauer lies at the surface as a transversely-placed flattened, or triangular band, with the narrow base directed laterally. Between Lissauer's marginal zone and the underlying substantia gelatinosa is a layer of coarsely myelinated fibres, of varying depth and compactness in the different species, but always showing strands of gray matter connecting the superficial part of the substantia gelatinosa with the deeper part of Lissauer's zone. SINN (1913) believes this layer to consist, in birds, of descending root fibres of the fifth cranial nerve. Lower down the cord, its existence can probably be explained by the presence of spinal root fibres, ascending and descending. It occurs also, though less marked, in man.

In general, on closer examination, five layers can be distinguished, on passing from the surface of the cord to the body of the horn. At the surface, Lissauer's zone contains a very thin layer of finely myelinated fibres: deep to this is a more open layer in which the fibres are fewer, and with

which there is a considerable mixture of gray matter. This gray matter is continuous, through the layer of coarse fibres, with the underlying substantia gelatinosa. The coarsely myelinated fibres can be divided into two zones, the more superficial, in general, being closely packed, though in some species (crocodile, fig. 4) more closely than in others (Dammonia). The fourth layer consists of an admixture of isolated bundles of coarse fibres and substantia gelatinosa. The deeper part of the substantia gelatinosa, which constitutes the fifth layer, is practically devoid of myelination, and resembles the substance in mammals.

The subdivision into five layers is well illustrated in the case of the *crocodile* (fig. 4).

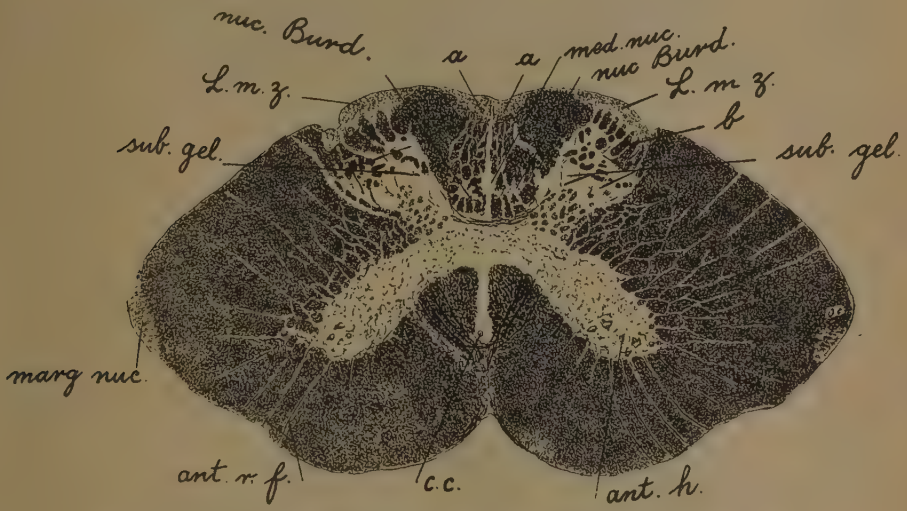

Fig. 4. Transverse section through the upper cervical region of the spinal cord of Crocodilus porosus $\times$ 18.

Regarding the recognition of two distinct areas in Lissauer's zone, SINN (1913) has described what I term the deeper layer of Lissauer's zone under the name "promontorium" in birds, and shows a close postural relation between its medial narrow extremity and the lateral nucleus of the posterior funiculus. ZEEHANDELAAR (1920) associates it in function with the posterior horn, and I think it more correct to regard it as part of Lissauer's zone, owing to the number of scattered, finely myelinated fibres it contains.

On tracing the series lower down the cord, the above relationship persists, but the entire horn narrows very much. Lissauer's zone becomes deeper and somewhat triangular, with the apex directed towards the centre of the cord. The zone of coarse fibres also narrows and deepens.

In fig. 4, at the surface of the cord, on either side of the posterior septum,

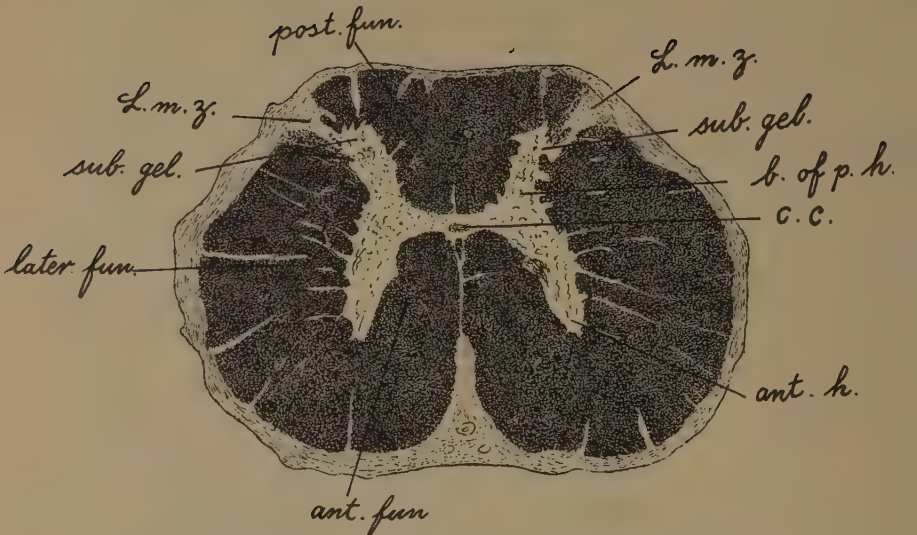

Fig. 5. Transverse section through the 2nd cervical segment of the spinal cord of Dammonia subtrijuga × 48.

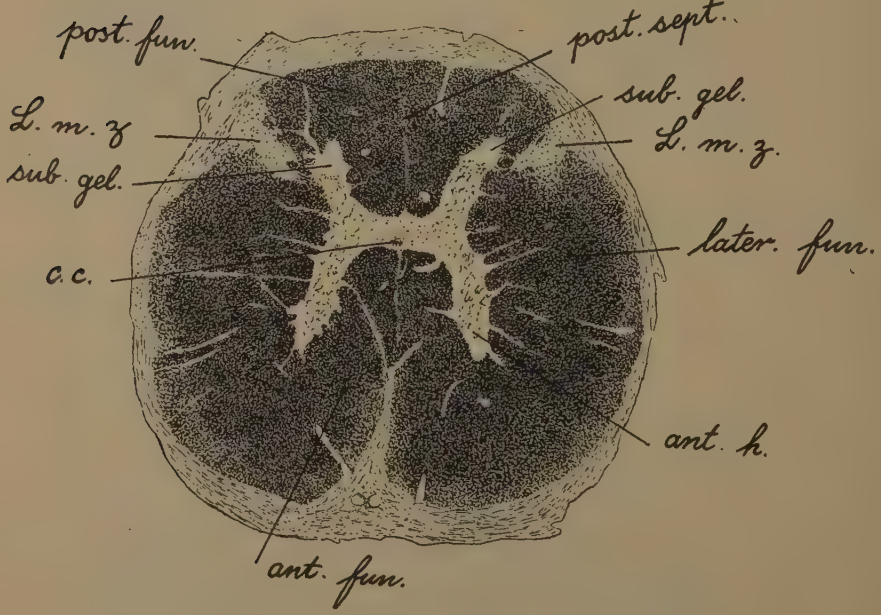

Fig. 6. Transverse section through the 4th thoracic segment of the spinal cord of Dammonia subtrijuga × 48.

we see an area, poorer in myelination than the rest of the posterior funiculus, and resembling Lissauer's zone very much in appearance. It is connected with Lissauer's zone bij a narrow sparsely-myelinated zone on the surface of the cord. It is probable, however, that this area is related in function with the underlying nuclei of the posterior funiculi, with which it is practically co-terminous.

In reptiles, there is a general tendency for Lissauer's zone to acquire a lateral position in relation to the rest of the posterior horn, and this is particularly marked in the case of Dammonia subtrijuga. At the upper end of the cord of Dammonia, Lissauer's zone is apical in position, but, lower down, it passes into a lateral position (figs. 5 and 6). In the mid-dorsal region, it becomes apical again, but gradually shifts laterally, and grows in size, as we approach the caudal end of the cord (fig. 7), until, in the lower sacral region, it lies in the concavity between the anterior and posterior horns. Intermingled with the fibres of this area are large nerve cells.

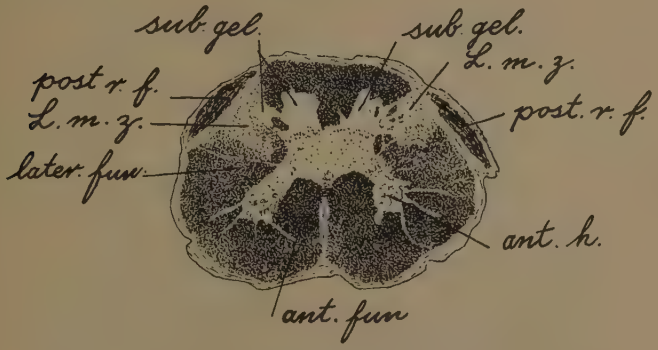

Fig. 7. Transverse section through the sacral region of the spinal cord of Dammonia subtrijuga × 48.

The lateral position of the zone in these forms may be another manifestation of KAPPERS' law of neurobiotaxis, as the posterior roots, on entering the cord, split into two bundles, one of which passes into the lateral funiculus. This bundle is strongly marked in snakes (cf. KAPPERS, 1920).

As far as concerns the gelatinous substance, it may be remarked, that it is also largest in the sacral region of Dammonia.

In snakes, Lissauer's zone is difficult to recognize on account of the number of closely packed fibres it contains. It is triangular in shape, with the apex narrowed to a point, and attached to the lateral aspect of the club-shaped head of the posterior horn. The posterior horns are united to form a horseshoe shaped structure, attached by a narrow neck to the anterior horns, as in birds (cf. fig. 8), and contain scattered masses of gelatinous substance.

Of birds, the following species were examined : — Ratites: Casuarius australis, and Struthio camelus.

Carinates: Athene noctua, Cacatua roseicapilla, Catharistes urubu, Ciconia alba, Columba domestica, Colynbus septentrionalis, Cyanistes coerulea, Cygnus olor, Gallus domesticus, Grus japonensis, Geranoaetus melanoleucus, Larus argentatus, Pavo cristatus, Podiceps cristatus, Sturnus vulgaris, and Spheniscus demersus.

In the upper cervical region of birds (fig. 8, Gallus), the same general relations as in reptiles exist. Lissauer's zone lies on the surface of the cord as a narrow triangular band, with the base turned laterally. The deeper part of the zone, to which SINN (1913) gave the name "promontorium", stretches medially almost as far as the posterior funicular nuclei, and consists of a mixture of gray matter and finely myelinated nerve fibres. Its lateral is wider than its medial end, and it communicates, around the lateral extremity of the layer of coarse fibres, with the gray matter of the posterior horn.

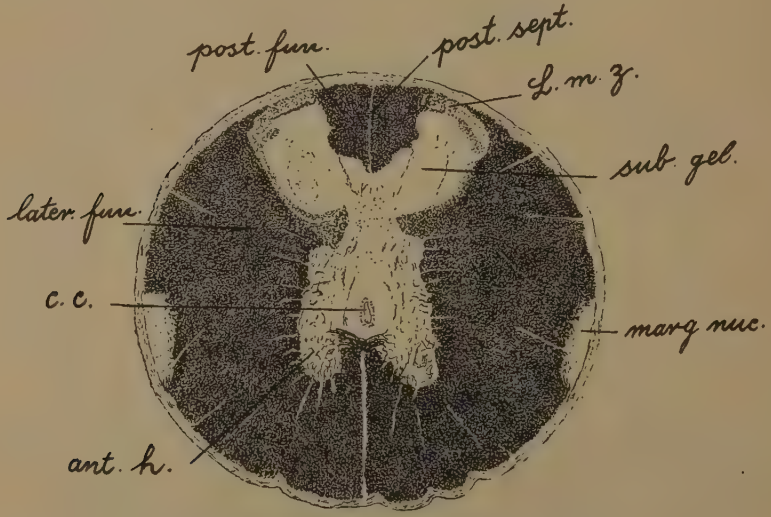

Fig. 8. Transverse section through the upper cervical region of the spinal cord of Gallus domesticus ×43.

Lower down the cord, the substantia gelatinosa is still prominent. Lissauer's zone, in addition to its apical position, extends down on either side of the posterior horn, and especially on the lateral side in the case of the pigeon. In the cervical swelling of the pigeon, the posterior horn is spearhead-shaped. The body of the horn, ventral to the substantia gelatinosa, is constricted, but widens again somewhat at its attachment to the anterior horn. Lissauer's zone is triangular with its apex at the surface

of the cord, and its oblique base rests on the lateral side of the substantia gelatinosa.

In the thoracic region, the substantia gelatinosa forms a club-shaped mass at the extremity of the body of the posterior horn, while Lissauer's zone has a similar relation to that in the cervical swelling.

In the sacral region the anterior horns greatly predominate in size over the posterior, which project laterally as narrow protuberances, in which gelatinous substance is still recognizable. As in amphibians there is no marked increase in the substance in this region of the cord, so typical of higher animals, and also indicated in Dammonia (fig. 7).

Fig. 8, taken from the upper cervical region of the chick, shows the great development of the substantia gelatinosa at this level, where it completely fills the posterior horns. Lissauer's zone shows the same structure as in reptiles, but the layer of coarse fibres is very small, and the gelatinous substance nearly touches Lissauer's zone. The horns are united to form a horse-shoe shaped mass attached, by a constricted area of gray matter, to the anterior horn region, and resembling very much, in this respect, the appearance in snakes.

The posterior funiculi of birds are small, according to BROUWER (1915), and confirmed by KAPPERS (1920), forming only 7 % to 8 % of the total area of the white matter in the cervical region. In the same region in reptiles, the percentage is 13 %. BROUWER believed that the decrease in the posterior funiculi was only relative to the great increase in the ventro-lateral tracts, but KAPPERS found, on comparing the posterior funiculi with the gray matter, that the posterior funiculi of birds are actually small.

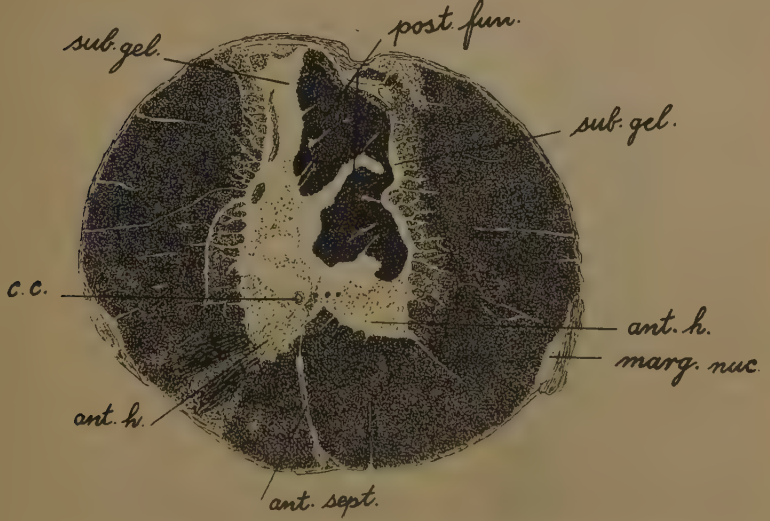

Fig. 9. Transverse section through the upper cervical region of the spinal cord of Larus argentatus × 25.

Many of the species examined did not conform to the description given. Whether these irregularities are of the nature of heterotopies, or are, more or less, normal in these cords, I cannot, at the moment, say. In a specimen of the *swan* (Cygnus olor), as we pass from oblongata to cord, the gray matter resembles that of Gallus domesticus in appearance, but, on tracing the series downwards, the posterior horns unite to form a single backward projection, consisting mainly of gelatinous substance, and connected with the surface, on either side, by an ill-defined quadrilateral Lissauer's zone. The posterior funiculi are small and devoid of a posterior septum. Only some upper cervical segments were available for study.

In a specimen of *Larus argentatus* (figs. 9 and 10), I found a very marked irregularity of the posterior horn region, apparently not normal. In the lower end of the medulla oblongata, and in the first segment of the cord, the relations conform to the general description given. Below this, the posterior funiculi become very irregular. The posterior septum is indistinguishable, and masses of gray matter invade and surround the funiculi. The funiculi extend ventrally, in an irregular manner, into the anterior horn region, until an appearance like that in fig. 9 is produced. On tracing the sections caudally, the posterior funiculi become smaller and finally disappear, the gray matter having an arrangement more or less similar to that shown in fig. 10. Further caudally, in the gray matter at the periphery of the cord, there appears a bundle of fibres, which increases in size until it almost cuts off the club-shaped ends of the posterior horns from the rest of the gray matter. It diminishes in size again, and ultimately disappears (fig. 10).

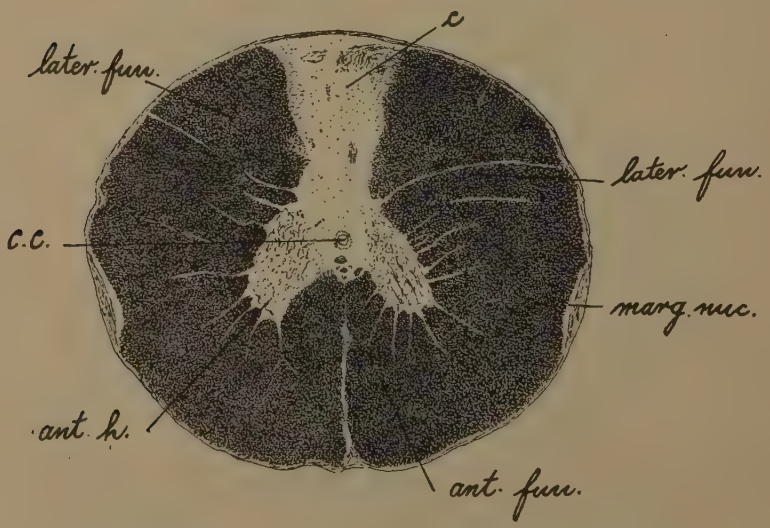

Fig. 10. Transverse section through the upper cervical region of the spinal cord of Larus argentatus, a short distance caudal to fig. 9, ×25.

Just above the point of disappearance of the funiculi, a posterior root enters the cord.

A few sections caudally, two bundles of fibres appear in the gray matter at the surface of the cord, and grow in size and coalesce, continuing to grow until very slender, irregular posterior horns are produced. Here the available series ends. The greater part of the gray matter consists of irregularly-scattered gelatinous-looking material, fringed by finely myelinated fibres. The above changes take place over a range of about five hundred 20 μ sections.

A somewhat similar phenomenon was observed in a specimen of the little owl (Athene noctua), and also in a specimen of Geranoaetus melanoleucus.

Résumé.

In *amphibians*, we find the first indication of a recognizable marginal zone of Lissauer. In fishes, fine and medium sized myelinated fibres are scattered through the gray substance of the posterior horn, and on either side of it, but their accumulation to form a single apical tract, such as exists in man, was not found. In the frog (Rana catesbyana), the fibres appear in the apical gray matter of the posterior horn as a widely separated but, nevertheless, recognizable tract. The substantia gelatinosa is on the whole poorly developed, but is present throughout the length of the cord, and extends from the apical area of fine fibres to the median septum in tranverse sections, and is broken up into nuclear-like masses by coarse fibres. There is a gradual reduction in amount on tracing the sections caudally. The lumbo-sacral region of the cord does not show the marked increase in substantia gelatinosa which occurs in reptiles and mammals, or even so definite an increase as in Albula vulpes.

Reptiles show a marked increase in development of Lissauer's zone. The superficial part of the zone is more compact in structure than the deep part, which contains much gray matter and few, widely separated fibres. A layer of coarsely myelinated fibres intervenes between Lissauer's zone and the underlying substantia gelatinosa. This layer is incomplete in places, allowing a continuity between the gray matter of Lissauer's zone and that of the superficial part of the substantia gelatinosa. The deeper part of this layer of fibres pervades the superficial part of the substantia gelatinosa as small isolated bundles. This gives the appearance of a separate zone, consisting of small bundles of coarse fibres mixed with substantia gelatinosa. The deeper part of the substantia gelatinosa is more compact, and surrounds, posteriorly, the extremity of the body of the horn, as in mammals. In crocodiles, the substantia gelatinosa spreads down along the medial side of the horn, almost as far as the median septum. Caudally, in the sacral region, the substantia increases in amount as does also Lissauer's zone.

Birds show well developed substantia gelatinosa. Most species examined conform to the general type in reptiles, resembling, especially, the cord of snakes. They do not, however, show a marked increase in the sacral region.

Some irregularities were met with, the significance of which it was thought advisable not to attempt an explanation of at present.

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Histology. — *The potencies and reactions of mesenchyma in fowls, in connection with the problem of avian leucosis.* By J. W. DUYFF. (From the Histolog. Laboratory of the University of Amsterdam, Dir. Prof. Dr. G. C. HERINGA.) (Communicated by Prof. J. BOEKE.)

(Communicated at the meeting of March 23, 1929).

Since 1845, the year in which VIRCHOW gave his classical description of human leucaemia, "leucotic" processes in animals, i.e. diseases showing a more or less superficial resemblance to the leucaemia, have been subject to many researches by various investigators, and a great number of such 'leucoses" have been described. Now the human leucaemia is a circumscribed, well-defined disease, with typical anatomo-pathologic features, with typical alterations in the circulating blood, both quantitative *and qualitative*, and always fatal. Before concluding that there is an analogy, and a fortiori a certain identity of one of those leucoses with human leucaemia, the presence in this leucosis of the typical features of leucaemia should be ascertained.

One of those so-called parallels to our leucaemia is the leucosis in fowls, as studied and described by ELLERMANN and BANG, by HIRSCHFELDT and JACOBY, and others, and which includes "leucaemia", "pseudoleucaemia", "anaemia". ELLERMANN, who published a monograph on the subject in 1918, considers this disease as a specific infection with a filtrable virus, contained in the spleen, the liver, the blood and the bone-marrow of the affected animals; about 6—8 months after injection of emulsions of the above organs or the blood of leucotic fowls into healthy ones, some 40 p. 100 of the latter fall ill; after injection of filtered emulsions about 20—25 p. 100. In one and the same series, however, the distemper can show itself in very different ways, so that alternately intravascular myeloid resp. lymphatic "leucosis", the corresponding extravascular (i.e. aleucaemic) processes, disseminated solitary lymphomata and anaemic cases can be found. When we bear in mind that ELLERMANN himself states the difficulty in differentiating some of his cases from cases of tuberculosis, — that in his "Versuchsprotokolle" a case with 43.000 leucocytes, and without definite qualitative alterations [1]) of the white corpuscles (young and unripe stadia cannot be considered as being abnormal; they can be found in every non-

[1]) Pronounced qualitative alterations of the white corpuscules would be much more important for the diagnosis of a genuine leukaemia, than even rather important numeral changes. The "pathologic" forms as described by ELLERMANN (i.e. the forms observed in cases of his leucosis), can be considered as juvenile stadia (grand lymphocytes = lymphoid haemoblasts; polychromatophile erythrocytes etc.)

specific hyperleucocytosis), is announced as an intravascular leucosis (the normal number of leucocytes varying from 23.000—36.500) ; — that the anatomo-pathologic alterations in the organs consist principally of an increase of the accumulations of leucocytes (myelocytes and lymphocytes) in the periportal spaces of the liver, in the kidney, etc. and of the so-called leucostasis (i.e. an accumulation of white blood corpuscles in dilated capillaries) ; — that in a great number of cases only these lymphatic resp. myeloid hyperplasia can be found, the circulating blood being normal in a rather considerable number of the cases described, and in the others showing mostly but rather insignificant alterations, — then it will be clear, that investigators should be guarded in statements about a parallelism of this distemper and the human leucaemia and about a specific infectious morbific agent of the leucosis, all the more so, because the haemopoietic system in hens is rather labilely balanced, and might be influenced in some way by non-specific agents. Care should be taken not to identify a well-defined disease and a myeloid resp. lymphatic reaction, a syndrome.

In 1916, DANTCHAKOFF published some very interesting facts about the potencies and reactions of the mesenchyma of the chick-embryo and of the grown-up hen. She grafted splenic tissue of adult hens on the allantochorion [1]) of chick-embryos of about 8—12 days of incubation and studied the local reactions as well as the alterations in the tissues of the embryo. Very briefly resumed, her results were the following :

a. *Local reaction.* Hypertrophy of the mesenchyma of the allantochorion with change of mesenchymal elements into blood stem-cells (so-called lymphoid haemoblasts) and thus into "granuloblasts" resp. "histiotopic wandering cells". Necrosis of part of the grafted tissue, migration of most of the small lymphocytes and erythrocytes contained in it into the allantoic tissue, with subsequent necrosis of these elements, granuloblastic differentiation of the reticulum of the adult splenic tissue itself.

   Slight hypertrophy of the ecto- and entodermal epithelia, considered as due to a local and mechanical agent, viz. the hypertrophy of the allantoic mesenchyma.

b. *Reaction in the embryo.* Myeloid metaplasia of the mesenchyma, enlargement of the spleen, with granulo- or erythropoiesis.

The results of these very interesting experiments allow of the following conclusions :

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[1]) The allantochorion is the membrane, resulting from the growing together of the allantois and the amniogenic chorion (séreuse de von Baer of the French). It consists of a vasculated mesenchyma, covered at the outer side with an ectodermal epithelium, originating from the chorion, and at the inner surface with an entodermal, allantoic epithelium. Under the ectodermal epithelium lies a tight respiratory capillary-plexus. (DANTCHAKOFF).

1. In the products of the mesenchyma of the chick-embryo neither the functional adaptation and specialisation, nor the resulting morphologic alterations are necessarily attended by a limitation of their potencies.
2. The loose mesenchyma in the chick-embryo, as well as the reticulum of the adult splenic tissue in fowls are polyvalent (cf. in this connection LAGUESSES conception of the spleen as a "reliquat du mésenchyme embryonnaire").
3. In BRACHET's terminology, the haemopoiesis is a "différentiation provoquée".

When comparing the results of DANTCHAKOFF's experiments with the features of ELLERMANN's leucosis, we find a striking resemblance. Of course it should be remarked, that DANTCHAKOFF has been able to keep the embryos livng only up to the 18—19th day of their praenatal development. As a matter of fact, this made thorough examinations of the circulating blood impossible, (apparently basing her conclusions on the features of the blood in her coupes, DANTCHAKOFF describes it as having the properties of myeloid leucaemia, without giving further particularities), nor could the further course of the reaction be examined.

In repeating, and in several points extending DANTCHAKOFF's experiments we had a threefold purpose: first of all, to verify her results, then to isolate, if possible, the agent causing the myeloid reaction, and in the third place to form an estimate of the relation between her results and those of ELLERMANN. A preliminary statement of the results obtained may follow here.

A. On the whole we can corroborate DANTCHAKOFF's conclusions; in some points only, more especially concerning the adhesion of the grafted tissue to the allantochorion and the local reaction of the latter we obtained a somewhat different result, so that we need but give a short description of our results, only emphasizing the points in which we cannot completely agree with DANTCHAKOFF's views. Briefly, our results can be summed up as follows:

   a. adhesion of the graft to the allantochorion.
   In this respect, we can fully confirm DANTCHAKOFF's exact observations. The ectodermal epithelium is arroded and interrupted by the grafted tissue, some parts of this epithelium lying isolated in the stroma, and partly changing into something like cornified pearls. Attention should be drawn to the fact, that in some of the cases erythrocytes from the grafted tissue were found lying against the ectodermal epithelium, and dinting it in as it were; afterwards the epithelium cells take up the necrotic erythrocytes; the enlarged cells themselves show phenomena of vacuolisation and degeneration. In several cases.

we found an epitheloid extension of erythrocytes over the ectodermal surface of the allantochorion [1]).
(The erythrocytic character of these cells has been ascertained by means of the benzidine staining.)

b. local response of the allantochorion.
General state of the membrane. — The area surrounding the graft shows a remarkable thickening of the whole allantoic membrane (caused principally by the alterations in the allantoic stroma, partly, but in a less degree by the epithelial changes), and is characterized by a formation of crypts and papillae on the entodermal surface; these alterations attain their maximum opposite the graft.

1. The *mesenchyma* shows in many cases, according to DANTCHAKOFF's results, an intense "myeloid metaplasia": (in some cases this metaplasia presents itself as a diffuse process, mostly we find a diffuse alteration of the mesenchyma, attended by very intense haemopoietic foci in the perivascular areas; in these perivascular haemopoietic foci we find lymphoid haemoblasts and an increasing number of granuloblasts, JOLLY's transitory cells, polynuclear "eosinophiles" [2]), and erythroblasts. — As a matter of fact, an interpretation of these phenomena as a myeloid metaplasia might be quite correct, though it is of course very difficult to exclude fully a myeloid infiltration.
2. The alterations of the *entodermal* epithelium, consisting principally of a stratification in about 5—6 layers of cells and the formation of papillous excrescentions, attain their maxium opposite the graft. Here we find mostly a real, solide, papillous epithelial tumor, consisting of fusiform elements, and in some cases showing central necrosis. Though the intensity of the entodermal reaction shows many variations, it is certainly not parallel to the extension of the mesenchymal hypertrophy, so that we cannot agree with DANTCHAKOFF's interpretation of the entodermal answer as being secondary and due to the hypertrophy of the allantoic stroma.
Moreover, in the results of further experiments (cf. B, II: β) we found some additional arguments for the conception of the reaction as a specific and independent process.

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[1]) In one case, part of the graft was attached to the egg-shell-membrane. Here a similar extension of erythrocytes was found.

[2]) These "eosinophiles", (or, better, pseudoeosinophiles), cannot be compared with human eosinophile granulocytes, but have perhaps a certain analogy with our neutrophile granulocytes; they form the majority of the myelogenic white elements in avian blood.

3. The *ectodermal* reaction cannot be separated from the process of adhesion of the graft to the allantochorion. Just as in the entoderm we find a hypertrophy and a stratification of the epithelium ; mitoses as well as amitoses are rather scarce [1]). In some cases the area near the periphery of the grafted tissue is characterized by the epithelium cells rankly growing into the stroma ; many of the cells leave the epithelial connection, but remain in relation with each other and with the epithelial layers by means of protoplasmatic expansions, and can only with difficulty be differentiated from mesenchymal elements.

c. **The alterations in the grafted splenic tissue** fully correspond to DANTCHAKOFF's description: small lymphocytes wander out of the reticulum and invade the mesenchyma of the allantochorion ; erythrocytes from the grafted tissue, too, can be found in the allantoic stroma; here erythrocytes as well as lymphocytes show necrosis, karyorrhexis and pyknosis. Part of the reticulum shows similar necrotic alterations ; the rest may be subject to myeloid metaplasia.

d. **Remote reaction in the embryo.** The intensity of the myeloid reaction of the embryonic mesenchyma shows many variations. About the factors influencing the extension of the myeloid metaplasia, cf. section B.

A summary description of a typical case may follow here :

Series I A N°. 2. Graft of splenic tissue on the allantochorion (10th day of incubation) result after 2 days. Zenker—Formol (Maximov), Dominici. The grafted tissue contains some necrotic erythrocytes with karyorrhexis, some small lymphocytes, and swollen reticulm-elements. Slight reaction of the ectoderm (hypertrophy, formation of some giant cells, and fusiform elements) many interruptions of the epithelium. Epitheloid extension of erythrocytes from the graft over the ectoderm.

Entoderm : proliferation, formation of crypts and rather plump papillae.

Mesenchyma : hypertrophy, hyperaemia, diffuse haemopoiesis.

Embryo : Diffuse myeloid metaplasia, slight enlargement of the spleen. The splenic tissue has lost its typical structure, and shows massive accumulations of myelocytes and eosinophile granulocytes In the periportal spaces of the liver and in the interstitium of the kidneys, many myeloblasts, myelocytes and eosinophiles are to be found.

B. In consequence of DANTCHAKOFF's experiments, some questions present themselves.

1) Here and there a pluripolar mitosis may be found.

I. *Is the reaction necessarily bound to the grafting of living tissue?* It is a well-known fact in human pathology, that necrosis is one of the most frequent causes of a myeloid reaction. In connection with this fact, the myeloid reaction in this case might be due to the necrosis of part of the grafted tissue, all the more so, because the remainder of the latter may be subject to a similar myeloid metaplasia itself. This view is borne out by the fact that the myeloid metaplasia is stronger in proportion as the necrosis of the grafted tissue is more important (the extension of the necrosis in the graft can be varied, e.g. by previously bruising part of it, etc.); (for the adhesion of the graft to the allantoic tissue, implantation of living tissue is required). In a case when instead of grafting tissue, a splenic extract was applied to the allantochorion, the general myeloid reaction was stronger than in any other case. This leads to the conclusion, that *the myeloid metaplasia is probably due to a chemical agent, proceeding from the decay of the cellular elements of the graft.*

II. *Is the reaction the specific result of the grafting of splenic tissue or of part of it?*

α. Grafting of different organs (liver, gl. thyreoidea, gl. suprarenalis, kidney) led to the conclusion, that the answer to every implantation is different and specific. A notable myeloid metaplasia is obtained by grafting of spleen, bone-marrow, and (in a less degree) of liver. The answer to the grafting of different tissues will be described circumstantially before long.

β. After grafting of splenic reticulum, previously washed out with sterile Ringer (in this case the grafts were subject to an extensive necrosis) the myeloid metaplasia was very marked; graftings of leucocytes and of erythrocytes, as well as application of extracts of these elements are in preparation. This experiment has some importance as to the interpretation of the epithelial hypertrophy; the massive myeloid metaplasia was but accompanied by epithelial reactions hardly worth mentioning. The epithelial reaction might be especially due to the products of the small lymphocytes; this would be in concurrence with Murphy's views about the relation between the autolysis of lymphocytes and epithelial proliferations.

β. Series VI A N°. 2α. Graft of washed-out reticulum (11th day of incubation).
Result after 7 days.
Zenker-Formol (Maximov), Dominici.

The *graft* consists of altered, partly necrotic reticulum-cells; no erythrocytes, an odd lymphocyte.
The *ectoderm* shows some very slender papillae, here and there a giant cell.
The *entodermal* reaction is hardly worthy of mention: flat epithelium without any trace of crypts or papillae. Some of the nuclei are small and dark, others enlarged and clear.
The *mesenchyma* is very dense and shows a pronounced angiopoiesis and extensive myeloid metaplasia; some of the myeloid foci are subject to central necrosis.
In the *embryo* the enlarged spleen contains but few erythrocytes; the normal structure of the organ cannot be recognized, the whole splenic tissue having been subject to an intense granulopoiesis. The periportal spaces in the liver show dense accumulations of granuloblasts; the renal interstitium is widened and contains many myelocytes.
The mesenchyma of the examined muscles shows a myeloid metaplasia. In the vessels, the number of granulocytes surpasses that of erythrocytes..

III. *Should the phenomena described be considered exclusively as an induced alteration of the elements of the embryonic mesenchyma?*

DANTCHAKOFF does not fully exclude the possibility, that in her experiments *invasion* of elements from the graft into the embryonic system and subsequent alteration of those elements might play a part. Now the results obtained by application of splenic extract, in which case there is no question about any invasion, show that the myeloid reaction need certainly not be due to any infiltration of the embryonic tissue by elements from the graft. Of course this does not exclude the possibility of invasion in the primitive experiments. At all events, if any invasion should exist, it is of very small importance. For the rest, there is another fact, which might tend to prove for an induced reaction; viz. the praedilective localisation of the myeloid foci round the blood-vessels.

We have tried to solve at least part of the question by application of previous vital staining of the grafted splenic tissue as well as of the allantochorion; only these experiments should be continued, as we have not yet obtained the results desired.

C. After many vain endeavours we obtained a series in which part of the setting matured [1]), so that repeated morphological blood examinations could be made, and the further progress of the reaction as well as its clinical features could be studied.

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[1]) This series consisted of 81 eggs, of which 16, (nearly 20 p. 100), matured.

I. *Clinical features.* A rather important part (7) of the chickens has expired already after having shown quickly aggravating symptoms; limp paralysis of the legs (Nos. 1, 2, 5, 7), ocular (N°. 2, 7?), and intestinal (diarrhoic) symptoms (Nos. 3, 5); in some cases ulcerating wounds (Nos. 4, 5), anaemia (Nos. 2, 4, 6, 7), progressive cachexia. As ELLERMANN does not describe any clinical observations as to his leucotic hens, a comparison with the features of his distemper could not be made.

II. *Anatomic alterations.*

a. Circulating blood: anaemia, in some cases a hyperleucocytosis up to 100.000 leucocytes with perhaps a slight relative increase (up to about 40 p. 100) of the myeloid elements; in several cases a leucopenia with about 2000—3000 corpuscles.

We did not find obviously abnormal granulocytes, any more than ELLERMANN or DANTCHAKOFF; of course young and unripe stadia were very frequent.

b. Alterations in the tissues. — The anatomo-pathologic features of our cases correspond with ELLERMANN's description; in the hyperleucotic cases as well as in the leucopenic ones, myeloid hyperplasia and metaplasia, solitary myelomata and myeloid infiltration of different organs, loss of the normal splenic structure with alteration of the elements of the splenic reticulum into erythroblasts resp. myelocytes and "eosinophile" granulocytes, and leucostasis may be found.

Some typical cases may be cited here:

Series D, N°. 5. Graft of adult splenic tissue on the 11th day of incubation. — Hatched Sept. 7th, 1928; died Nov. 5th.

Progressive limp paralysis of the legs, aggravating cachexia.

Blood-count: marked anaemia (2.300.000); many polychromatophile erythrocytes; an odd erythrocyte shows a mitotic figure.

Spleen: enlarged, loss of normal structure, diffuse myeloid metaplasia. — Liver: slight increase of periportal myeloid accumulations. — Intestine: foci of myeloid metaplasia in the submucosa.

Lungs: hyperaemia, dense accumulation of lymphoid haemoblasts, changing into granuloblasts and eosinophile leucocytes.

Series D, N°. 3. Graft of adult splenic tissue on the 11th day of incubation. — Hatched Sept. 7th, 1928; died Oct. 1th.

Progressive cachexia, intermittent paralysis, diarrhoea.

Blood-count: erythrocytes 4.000.000; white cells 97.500; lymphocytes about 35%; marked increase of the number of myeloid elements; many unripe stadia.

Spleen: hyperaemia, myeloid metaplasia. — Kidney: widened interstitium with disseminated small accumulations of myeloid elements.

Liver and intestine: hyperaemia.

The material collected is too small still to allow of any definite conclusion as to the further course of the reaction; the course of part of the cases with quickly aggravating symptoms, leading to a rather early exitus might point to a certain progression, as no defined intercurrent distemper could be made responsible for the symptoms described.

Lastly, the question arises, whether the reaction described could be carried over to other animals. — Using the technique as described by ELLERMANN, sterile emulsions of spleen and liver of the deceased animals were made, and intravenous injections of these emulsions in 6 adult hens were performed; in 4 chickens intraperitoneal injection was used. The time elapsed since is too short to allow of a conclusion as to the appearance or non-appearance of alterations responding to ELLERMANN's description (in one of the cases a hyperleucocytosis of about 50.000 could be observed). Especially in the cases, where adult hens were used, a marked reaction might seen a priori rather improbable, as the products of the embryonic mesenchyma in the adult hens might have lost part of their potencies as a consequence of their further differentiation. On the other hand, DANTCHAKOFF's experiments show clearly, that in adult hens the mesenchyma of the spleen at least is still polyvalent; moreover, as we observed already, the equilibrium of the haemopoietic system in hens is a rather labile one. In this connection it is important to observe, that the virus of ELLERMANN's leucosis is said to be contained exactly in the organs normally connected with the processes of blood formation and blood decay, and by decay of which, in accordance with DANTCHAKOFF's and our experiments, substances causing a myeloid metaplasia of the embryonic mesenchyma as well as of the adult splenic reticulum in hens, are produced.

For the rest, a positive result would mean nothing as to the etiology and the pathogenesis of the spontaneous avian leucosis; it would be merely another warning to be guarded in admitting the existence of a specific infectious agent causing it; another argument not to conclude that there is an anology with human leucaemia, if this conclusion is based merely on a more or less superficial resemblance; — another argument lastly, not to transfer considerations about distempers in animals to "similar" diseases in man, without having serious and conclusive arguments for it.

EXPLANATION OF ILLUSTRATIONS.

Fig. 1. General view of the allantochorion of a chick embryo, 3 days after the grafting of adult splenic tissue on its ectodermal surface. Hypertrophy of the allantoic membrane, complete adhesion of the grafted tissue to the allantochorion, slight ecto- and

entodermal reaction, beginning reaction in the allantoic mesenchyma (tightening of the stroma, hyperaemia).

Fig. 2. Allantochorion of a chick embryo, 2 days after the grafting of adult splenic tissue. Small perivascular haemopoietic focus.

Fig. 3. Allantochorion of a chick embryo, 3 days after the grafting of adult splenic tissue. Marked reaction of the entodermal epithelium; formation of papillous excrescentions, tightening of the allantoic stroma in the corresponding area; hyperaemia.

Fig. 4. Allantochorion 3 days after grafting. Typical papilliform excrescentions, vacuolisation of the elements, beginning of central necrosis.

Fig. 5. 2 days after grafting of adult splenic tissue. Formation of crypts and papillae on the entodermal surface of the allantoic membrane, hypertrophy and stratification of the entoderm.

Fig. 6. From the allantochorion of a chick embryo (entodermal side), 2 days after a particle of washed-out adult splenic reticulum has been grafted. Intense diffuse myeloid metaplasia of the allantoic mesenchyma, attended by a minimal answer of the entodermal epithelium.

Fig. 7. From the liver of a chicken, died 24 days after hatching. On the 11th day of its praenatal development adult splenic tissue was grafted on the allantochorion Periportal myeloid metaplasia: myeloblasts, myelocytes, pseudoeosinophilic granulocytes.

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5. ————————, The leucosis of fowls and leucaemia problems. London 1921.

(This is a preliminary report.)

Fig. 1. (A*. Zeiss; peripl. oc. 4×).

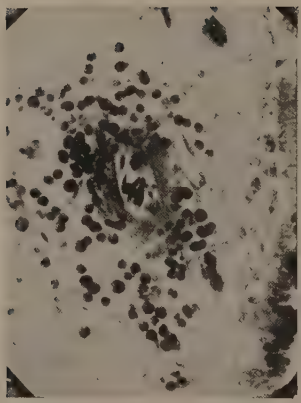


Fig. 2. (Achromate 7 mm. Leitz; peripl. oc. 4×).

Fig. 3, (Apochrom. 16 mm. Leitz; peripl. oc. 4×).

Fig. 4. (Apochrom. 16 mm. Leitz; peripl. oc. 4×).

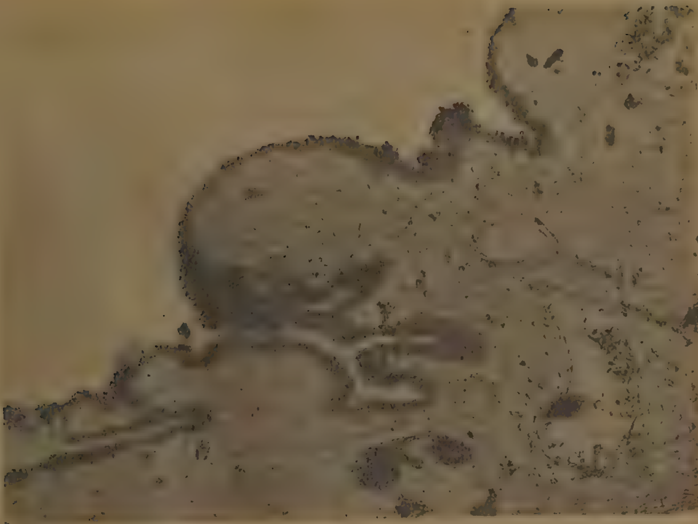
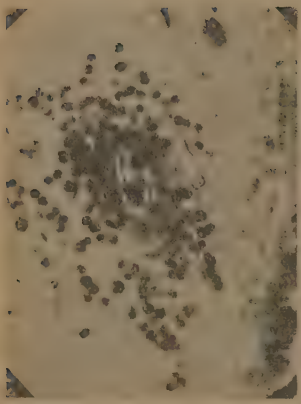


Fig. 5. (Apoc[illegible] mm. Leitz: peripl. oc. 4 ×).

Fig. 6 (Apochr. 8 mm. Leitz: peripl. oc. 4 ×).

Fig. 7. (Apochr. 8 mm. Leitz: peripl. oc. 4 ×).

Fig. 1. (A*. Zeiss; peripl. oc. 4×).



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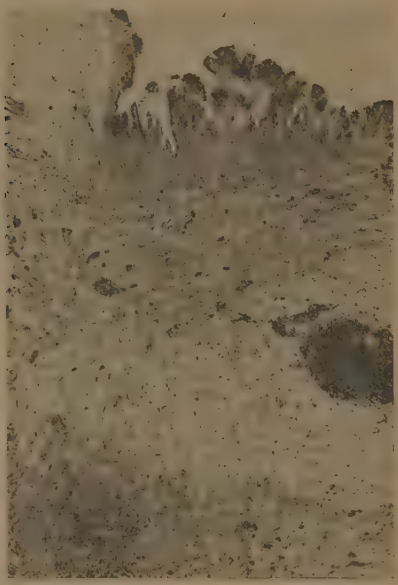
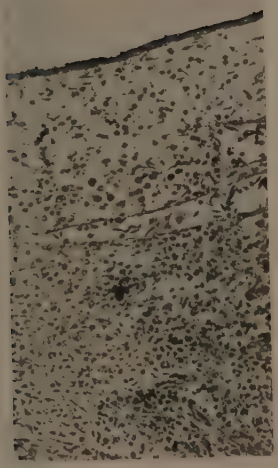


Fig. 3, (Apochrom. 16 mm. Leitz; peripl. oc. 4×).

Fig. 4. (Apochrom. 16 mm. Leitz; peripl. oc. 4×).

Fig. 5. (Apochrom. 16 mm. Leitz; peripl. oc. 4×).



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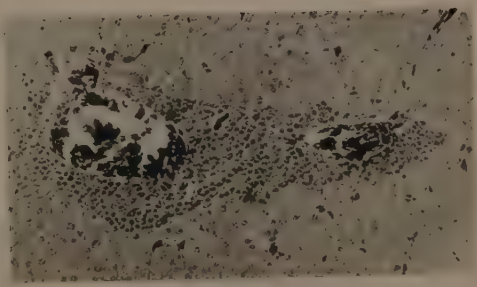


Fig. 7. (Apochr. 8 mm. Leitz; peripl. oc. 4×).

Palaeo-Zoology. — *On the Oldest Domestic Animal and its significance for Palethnology.* By A. E. VAN GIFFEN. (Communicated by Prof. J. F. VAN BENMELEN.)

(Communicated at the meeting of December 17, 1927).

This paper records the first results of a lengthy and time-consuming inquiry regarding pre-, and proto-historical domestic animals, notably dogs, which, at all events, in Europe, seem indeed to be the oldest companions of man.

One of the earliest known domestic animals is *Canis palustris*, acknowledged as such by RÜTIMEYER. Besides this we have since that time gathered much information concerning other prae-, and proto-historical forms from the works of JEITTELES, WOLDRICH, STROBEL, TROUESSART, STUDER, KELLER, WINGE, HILZHEIMER, BRINKMAN, etc., etc. I mention only:

Canis palustris ladogensis Anutschin
Canis inostranzewi Anutschin
Canis matris optimae Jeitteles
Canis intermedius Woldrich
Canis mikii Woldrich
Canis spalletti Strobel
Canis de le Mirei Hue
Canis leineri Studer
Canis putiatini Studer
Canis intermedius newelskii Brauner
Canis kryschtofovitschi Brauner
{ Canis decumanis Nehring } (Roman)
{ Canis molossus Kraemer }

When at the time of the commencement of my investigation of the "Terpen" [1]) in the year 1908 I was confronted with a large number of proto-historical remains of domesticated animals I was desirous to examine the material by a similar method to that adopted by RÜTIMEYER in his research of the fauna of the Swiss lake-dwellings. This not only facilitated my efforts, it also placed at my disposal a number of so-called standard types. But at the time the result of those efforts consisted only of a thesis entitled "Die Fauna der Wurten". The title is disappointing, as it suggests much more than the contents impart, so the addition "Teil I" on the title page is everyway justifiable. However, the fact is that "Teil II" has never

[1]) "Terpen" = the artificial protohistorical hills in the low lands along the shores.

appeared, although I may apologize by adding that I have been busy at it, and have already gathered some material for it.

It is into this material that I wish to make a dip now.

How was it that the study of the domesticated "Terpen"-fauna was retarded so long, in spite of the masterly work done by RÜTIMEYER, and notwithstanding the establishment of the above-named standard-types, and even despite the increase of our knowledge of domestic animals. It stands to reason that various factors may have come into play here, as e.g. pursuits in another direction, irrelevant private circumstances, difficulties in appreciating osteometrical researches, arising from the altered views in the genetic domain, etc., etc. It would not do, however, to ascribe this neglect to lack of interest. The contrary is the fact.

The true and primary cause of the retardation of the results of the early specific study of domestic animals must be looked for in the overwhelming mass of avaible material and the doubtful significance of the standard-type, as well as in the selection of the special domestic animal for a first treatment, i.e. the dog.

Before long I had the disposal of a few hundreds of skulls from a large variety of dogs, not to mention other skeletal parts. The standard-types mentioned appeared not to be adequate for the determination of these bones. Moreover the border values of the latter, and the features considered as typical of them, appeared to give rise to the greatest difficulties.

All sorts of differences between those types could be removed by help of that new material, whereas a number of forms lay quite beyond the limits assigned by them.

All this, added to the knowledge that identity of phenotype does not warrant identity of genotype, made me look out for another working-method.

A lecture by Prof. J. W. MOLL on Darwinism and experimental systematics in 1909 induced me to study the material systematically, in which efforts I was constantly encouraged by Prof. J. F. VAN BEMMELEN. I feel specially indebted to him for enabling me to carry out a new investigation of the Swiss lake-dwellings.

We refrained from a determination of the available forms piecemeal, and worked the material "en masse", i.e. the population as a whole, i.c. a group of domesticated animals, typical of the first half of our era. It seemed to me that it would be interesting for the solution of questions about origin and affinity, if we could work and compare such groups, as they are also characteristic of other regions and of other times. Therefore I will make a dip into the studied material, for if this working-method and the conception underlying it, should be right, it might just as physico-anthropological, prehistoric-archaeological, philological and other methods do, furnish an independent series of supplemental and also partly new data for an insight into the palethnological problems. If I am not mistaken this has already been shown.

A comparative, and sofar as was possible, a statistic study was made of the results of our investigation and measurement of three large populations of dogs and of one smaller one, viz.

a. from the Frisian and Groningen "Terpen" of the late iron-age ;

b. from the neolithic, and bronze-age Swiss lake-dwellings ;

c. from the mesolithic Danish-Cimbric Kjøkkenmøddingen, and

d. from the neolithic-megulithic settlement of Flintholm in the island of Alsen.

For the sake of comparison we also studied recent dogs, as well as their recent and less recent relations, to be found exclusively in the neighbourhood of Thooiden, the wolves and the jackals, from various regions.

Broadly outlined the results are provisionally to the following effect :

1. The terp-dogs inter se display much smaller divergencies than the recent dogs. A later increase is quite ascribable to the bulldog- resp. pug-type on the one side, and the greyhound on the other. The indices imply a predominence at the time of large bulldogs, hounds, and the like in our parts. Huge, greyhound-like forms of Leineri-, resp. "Deerhound"-type, were, however, represented among them. They betray much wolf's blood.

2. When compared with the dogs from the lakedwellings the Terp-dogs reveal an enormous increase of divergencies. The lacustrinc dogs themselves can be split up into two groups, (double-topped curves with more than fifty characteristics). They are a large group of smaller dogs, and a small group of larger dogs. To the former belong C. palustris, C. de le merei, and C. spalletti, to the latter C. matris optimae, C. leineri, C. inostranzewi, while C. intermedius is to be grouped under the latter for one characteristic, and under the former for another. Furthermore it has appeared that the small group of larger dogs is absent in lakedwellings with pure stone-cultures, while the palustris group characteristic of it shows fewer variations, or rather smaller differences than those of the lake-dwelling-culture, as a whole. For the rest as regards the facioneurocranium-length-index, and the neurocranium-width-index, the small group behaves like the jackals, but the large group like the wolves.

3. The Kjøkkenmøddinger dogs when compared with the dogs from the lakedwellings collectively, even with those from the some-age, exhibit on the contrary a considerable decrease of divergencies. Apart from the possibility that they also admit of a division into two groups, just as the arctic dogs, their divergencies agree completely with those of the palustris group of the lake-dwelling of the stoneage. They are isomorphous inter se, and on an average not only larger than the latter, but also larger than the well-known European jackal-forms.

4. Compared with the dogs from the lake-dwellings, and with the Kjøkkenmøddinger-dogs the Cimbric-megalithic dogs bear a great resemblance to the latter.

To express this more distinctly I may be allowed to report here a single

variability-coefficient, more particularly "Streuungs"-coefficient, namely $C = 100 \times \frac{\sigma}{M^a}$. In this way we can demonstrate more or less the divergencies that increase gradually with age. The coefficient in question is namely for the length of the praemolars (III—IV = 27), the molars + cuspid (I—III = 28), the molars (II—III = 29), and of the cuspid ($PM\frac{4}{4} = 56$) superior, resp. for those of the base of the jaw (ba = 62), the row of molars (III—IV = 66), the three molars (I—III = 67), the two posterior molars (II—III = 68), and of the cuspid ($M\frac{}{1} = 78$), for the above-named canine populations, successively:

Characteristic:	Dogs from the			
	"Terps"	Lakedwellings		Kjøkken-møddinger
		together	stone age	
III—IV = 27	$12.^{4}$	$8.^{2}$	$7.^{2}$	—
I—III = 28	—	$8.^{3}$	$7.^{5}$	$6.^{2}$
II—III = 29	11	$9.^{5}$	$8.^{4}$	$7.^{2}$
$PM\frac{4}{4}$ = 56	$10.^{2}$	$9.^{2}$	$8.^{5}$	$5.^{4}$
ba = 62	$17.^{4}$	$12.^{3}$	$8.^{6}$	$3.^{6}$
III—IV = 66	—	$12.^{5}$	$7.^{4}$	$4.^{2}$
I—III = 67	—	$9.^{4}$	$6.^{8}$	$5.^{7}$
II—III = 68	—	$12.^{6}$	$9.^{6}$	$4.^{5}$
$M\frac{}{1}$ = 78	—	10.	6.	$6.^{1}$

From the available comparable statistic results we may, to my thinking, conclude already now:

1. in particular:

a. that the Canis de le mirei of the French lake-dwellings is not a type of itself, but belongs to the palustric-group;

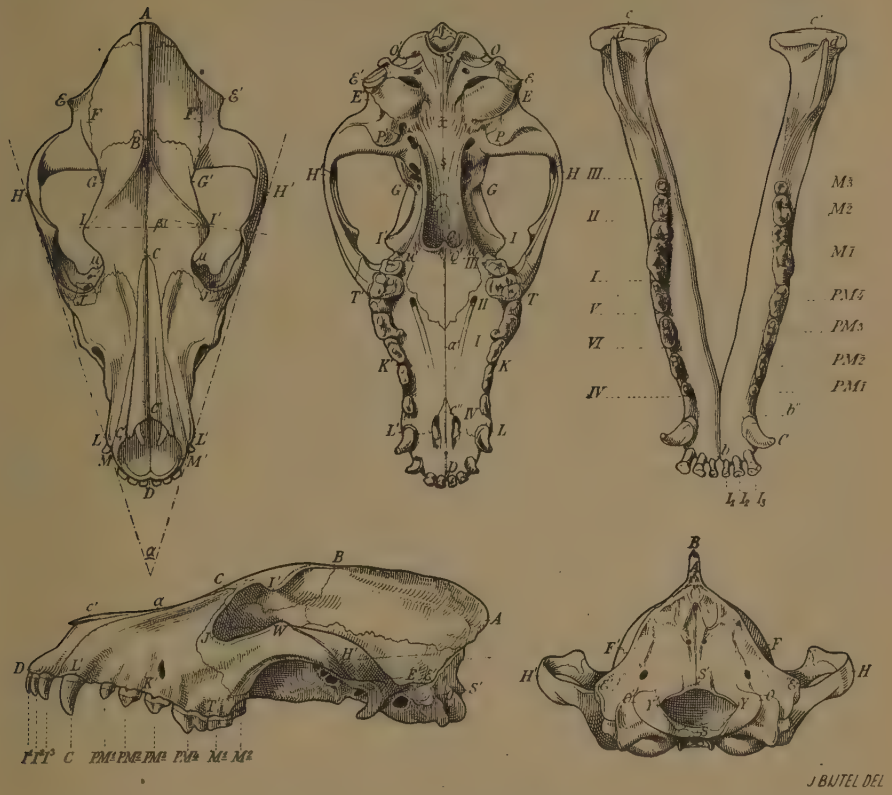
b. that the dog of the at all events typological, meso- and neolithic culture of Ryckholt—St. Geertruid, does not belong to the palustris-group, but to the group of larger dogs, which elsewhere do not appear before the lake-dwellings of the bronze age.

2. in general:

a. that e.g. earliest known Asiatic dogs (Anau in Turkestan) do not belong to the *palustris-*, but to the *matris optimae-* (wild form Canis pallipes Sykes) type;

b. that the earliest known European (i.e. Kjøkkenmøddinger) dogs

Localization of the measurements. PLATE 1



differ very little mutually; that they have nothing to do with the well-known analogous Asiatic or African dogs; that they do not descend from the Canis palustris of the lakedwellings; but that they may be the progenitors of the prae-historic, Cimbric-megalithic (and recent arctic) dogs, while conversely it is difficult to derive them from a European, neither to non-European jackals. They represent either a Canis verus, a "chien sui generis", or descendants from a small wolf of which we know nothing in particular.

The research for the place of origin, and the descent of these dogs must be of paramount importance for palethnology.

Whatever may be the significance of the objective facts memorized, one thing at which I hinted at the outset, is distinctly coming to the fore, namely, that it imports us to know what results would be yielded by a comparable statistic study of other than the prehistoric groups, named

Lakedwelling Dogs.

Characteristic $CC^1 = 3$
Length nasal bone

		Interval	Number	Frequency	Portion belonging to the pure stone-culture: Number	Frequency	Procentic number of all lacustrine dogs
●	Stone	44^5–47^4	5	5.55	5	10.87	5.55
✳	Copper	47^5–50^4	14	15.60	4	8.70	4.44
△	Bronze	50^5–53^4	19	21.10	13	28.26	14.46
● ✳	Stone-copper	53^5–56^4	20	22.20	9	19.56	10.00
● △	Stone-bronze	56^5–59^4	12	13.35	6	13.07	6.67
● ✳ △	Stone-copper-bronze	59^5–62^4	9	10.00	6	13.07	6.67
		62^5–65^4	5	5.55	1	2.17	1.11
		65^5–68^4	3	3.33	1	2.17	1.11
		68^5–71^4	1	1.11	1	2.17	1.11
		71^5–74^4	1	1.11			
		74^5–77^4	0	0.00			
		77^5–80^4	0	0.00			
		80^5–83^4	1	1.11			
	Total		90	100.01	46	100.01	51.11 %

155
149 148
128 140
123 139
104 127
91 121
111 88 120
110 73 114
95 70 108 122
94 69 102 112
87 67 90 107
68 65 64 103 A_4
17 63 50 97 150
16 49 42 89 142
12 48 29 62 137
156 9 44 26 38 99 A
147 8 30 25 37 45 125
105 3 22 7 31 39 100 E_8
41 2 18 6 23 27 92 C_1
36 1 11 4 10 20 75 A_2 C_3 C_2 D_1

44^5 47^5 50^5 53^5 56^5 59^5 62^5 65^5 68^5 71^5 74^5 77^5 80^5
47^4 50^4 53^4 56^4 59^4 62^4 65^4 68^4 71^4 74^4 77^4 80^4 83^4

	Group I	Group II	A
Minimum Variant	45	67	● 64
Maximum Variant	66	83	

	M	Q^o	Q^p	Q^1	Q^2	$\frac{Q}{M} = \frac{Q^1+Q^2}{2M}$
All together	54,55	51,05	58,87	3,50	4,32	0,072
Group stone age	53,67	51,08	58,25	2,59	4,58	0,067
Group C. palustris	54,17	50,82	57,75	3,35	3,58	0,064

	Minimum	Maximum
Group I	45	66
Canis aureus	45	56 (61)
Canis lupus	71	93
Group II	67	83
Terpdogs	23	93

x) E_1 K_2 E_2 A^1 A_1 D xx)
E_3 K_7 K_6

after Rütimeyer		Values found
Width of variation	47–58	45-66
Arithm. means	52^5 M	54,17

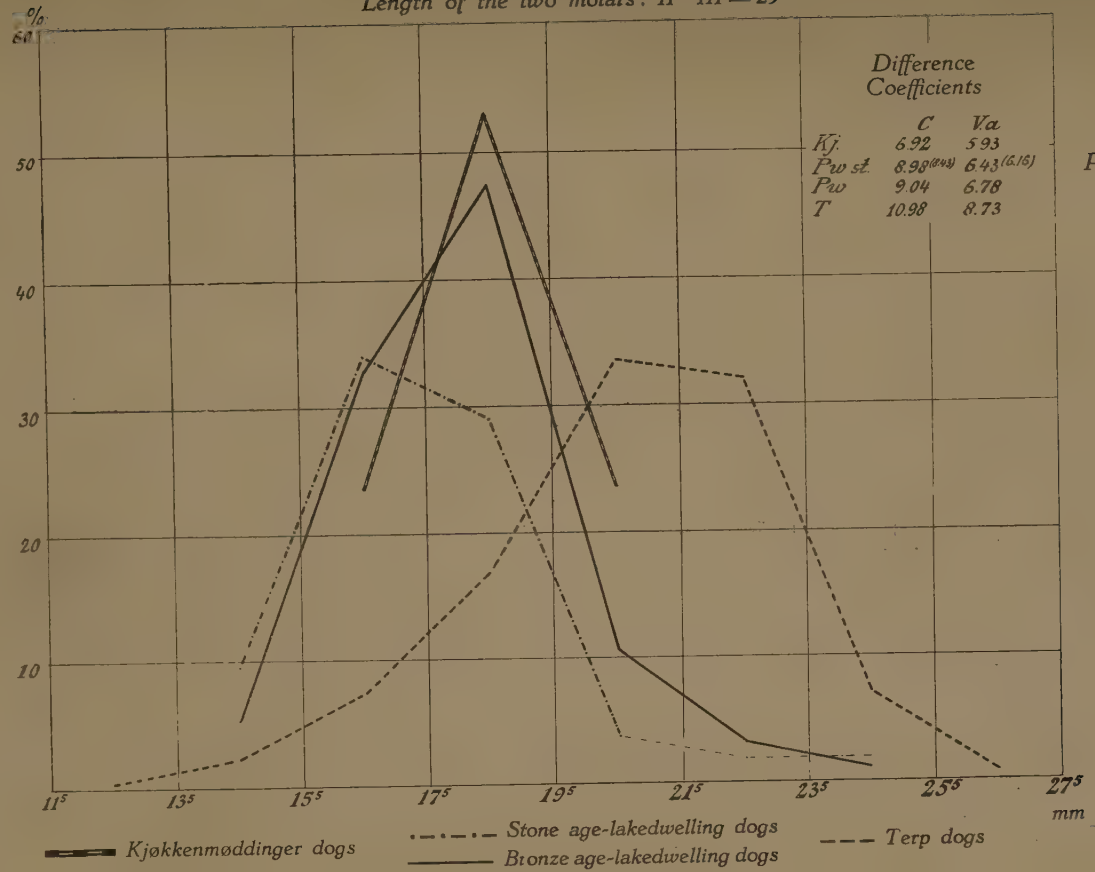
x) $K_1 = 43\ m\,M$

xx) $I_2 = 95\ m\,M$

N.B. Number [1]) with A = Canis matris optimae JEITTELES.
" " B^2) = " palustris RÜTIMEIJER.
" " C = " inostranzewi ANUTSCHIN.
" " D = " leineri STUDER.
Number with E = Canis intermedius WOLDRICH.
" " K = " spalletti STROBEL.
" " I = " decumanis NEHRING.

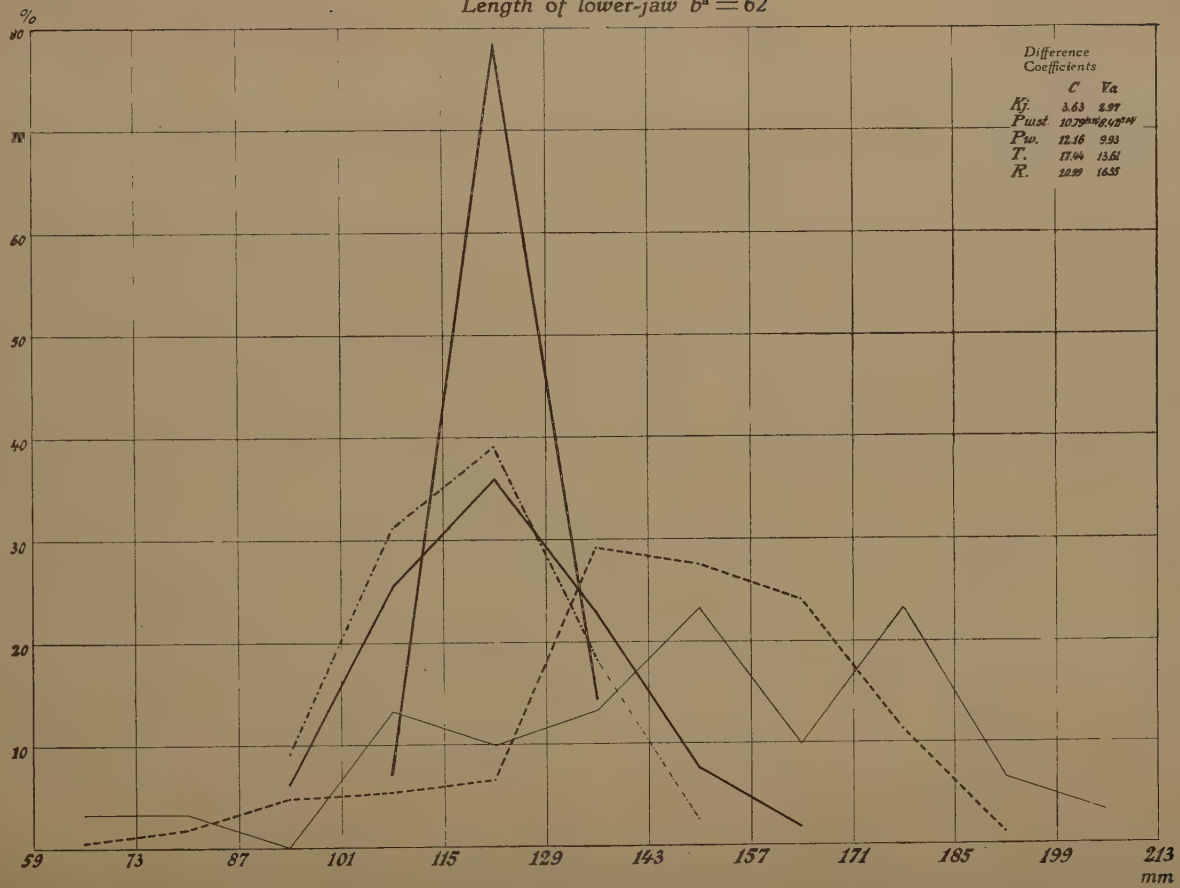
[1]) These numbers refer to those in the catalogue of the remains of dogs in the different Swiss collections as described or measured by the writer.

[2]) Since the greater part consists of Canis palustris, the latter B is omitted.


Frequency-curves
Length of the two molars: II—III = 29
%
60
50
40
30
20
10
11⁵
13⁵
15⁵
17⁵
19⁵
21⁵
23⁵
25⁵
27⁵
mm
Difference Coefficients
C
Va
Kj. 6.92 5.93
Pw st. 8.98(Ldw) 6.43(Ldw)
Pw 9.04 6.78
T 10.98 8.73
Pw.st. = Ldw.st.
Pw = Ldw.
Kjøkkenmøddinger dogs
Stone age-lakedwelling dogs
Bronze age-lakedwelling dogs
Terp dogs

Frequency-curves

Length of lower-jaw $b^a = 62$



Statistic comparative review of the characteristics: length of lower-jaw-, and tearing molars of the oldest housedogs and those of their nearest wild relations.

ba = 62

	94.5	101.5	108.5	115.5	122.5	129.5	136.5	143.5	150.5	157.5	164.5	171.5	178.5	185.5	192.5	199.5	206.5 mm
Jackals																	
South-Asia	2	12	8	1													
Minor-Asia	2	13	11	3													
Africa	1	14	12	7	2	1											
Europe (S.E.)	1	5	7	2													
Wolves																	
Europe											3	1	11	10	6	2	3
Diluvium Europe												2					
Asia								1		1	1	1					
Oldest dogs																	
Kjøkkenmøddinger			111		120⁴	123⁸	127⁵	132									
Stone age-Lakedwellings	95		108⁴		117²		126¹				157						
Peat-findings Denmark				1		1		2	6	5	5	8	2	number			

numbers

Explanation: — min. variant, — max. variant, — Q_a^0, = Ma, ≡ Q_a^p

Medians	
Kjøkkenmøddinger dogs	123.7
Stone age-Lakedw. dogs	117.9
Jackals: Europe	117.0
Jackals: Africa	117.0
Jackals: Minor-Asia	115.2
Jackals: Mesopotamia etc.	113.5

PM^4 = 56

	14.3	15.3	16.3	17.3	18.3	19.3	20.3	21.3	22.3	23.3	24.3	25.3	26.3	27.3	28.3	29.3 mm
Jackals																
South-Asia	4	7	10	3												
Minor-Asia	4	8	14	5												
Africa	3	4	12	12	4	3	1									
Europe (S.E.)		6	7	2												
Wolves																
Europe								2	5	4	6	9	7	3	1	
Diluvium Europe						(1)		1				2	4	2		
Asia							1	1	2	1	1					
Oldest dogs																
Kjøkkenmøddinger		15⁵	16⁰⁵	16⁹	17⁶		19²									
Stone age-Lakedwellings	15		16¹⁵	17²			18⁵		21							
Diluvial dogs																
Gaillenreuth. G 32						(20)										
Peat-findings Denmark			2	2	7	9	6	4	number							

numbers

Explanation: — min. variant, — max. variant, — Q_a^0, = Ma, ≡ Q_a^p

Medians	
Kjøkkenmøddinger dogs	16.9
Stone age-Lakedw. dogs	16.9
Jackals: Europe	16.5
Jackals: Africa	17.1
Jackals: Minor-Asia	16.5⁵
Jackals: Mesopotamia etc.	16.4⁵

M_1 = 78

	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5 mm
Jackals										
South-Asia		10	13							
Minor-Asia	8	8	18	3						
Africa	1	6	22	8	1	1				
Europe (S.E.)		9	5	1						
Wolves										
Europe							6	15	14	6
Diluvium Europe								1	6	8
Asia						1	2	2		
Oldest dogs										
Kjøkkenmøddinger			17⁸	19⁵	20⁴⁵	21⁴		24		
Stone age-Lakedwellings			17³	18⁶	19⁷		20⁸		22⁵	
Peat-findings Denmark				1		17		17		1 number

numbers

Explanation: — min. variant, — max. variant, — Q_a^0, = Ma, ≡ Q_a^p

Medians	
Kjøkkenmøddinger dogs	20.5
Stone age-Lakedw. dogs	19.8
Jackals: Europe	17.3
Jackals: Africa	19.1
Jackals: Minor-Asia	18.8
Jackals: Mesopotamia etc.	18.6

before, especially of Southern and Western Europe on the one side and Western Europe on the other. If we had comparable curves also of those types, I feel convinced that the solution of the palethnological problems would be largely benefited. It is my firm conviction that to this end it will be first of all required to make a careful collection, (too often neglected) of the often ignored prae-, and protohistorical bones of domesticated animals, and that international co-operation will be of inestimable value. My conviction is based on the statistic results obtained, especially curves. differences in width, medians quartiles and variation coefficients. I here refer to the diagrams and the tables. (Plate I—V.)

Mathematics. — *Allgemeine Räume und Cartesische Räume III* (Beweis des Fundamentalsatzes). By KARL MENGER. (Communicated by Prof. L. E. J. BROUWER).

(Communicated at the meeting of January 28, 1928).

Im Folgenden wird der in der ersten Mitteilung [1]) angekündigte allgemeine Einbettungssatz bewiesen: *Ein n-dimensionaler kompakter Raum ist homöomorph mit einer Teilmenge des* R_{2n+1}, (des $(2n+1)$-dimensionalen Cartesischen Raumes). Die Beweismethode lässt sich, wie ebenfalls in der ersten Mitteilung erwähnt wurde, ohne prinzipielle Schwierigkeiten auf beliebige separable Räume übertragen, und ergibt dann das Theorem: *Jeder n-dimensionale separable Raum ist homöomorph mit einer Teilmenge des* R_{2n+1}. In der zweiten Mitteilung [2]) wurde die viel weitergehende Vermutung ausgesprochen, dass jeder n-dimensionale separable Raum homöomorph sei mit einer Teilmenge *einer n-dimensionalen abgeschlossenen Teilmenge* des R_{2n+1}, ja sogar mit einer Teilmenge einer *bestimmten* (*"umfassendsten"*) n-dimensionalen abgeschlossenen Teilmenge des R_{2n+1}. Die in dieser vermuteten Behauptung enthaltene Aussage, dass jeder n-dimensionale separable Raum homöomorph sei mit einer Teilmenge eines n-dimensionalen kompakten Raumes, wurde inzwischen von HUREWICZ [3]) bewiesen. Auch aus dieser letzterwähnten Tatsache zusammen mit dem hier bewiesenen. Einbettungssatz für kompakte Räume folgt der Einbettungssatz für separable Räume.

Das einzige dimensionstheoretische Hilfsmittel, welches im Folgenden zum Beweis des Einbettungssatzes herangezogen wird, ist mein allgemeines Zerlegungstheorem [4]): *Ein n-dimensionaler kompakter Raum ist für jedes* $\varepsilon > 0$ *Summe von endlich vielen abgeschlossenen Mengen mit Durchmessern* $< \varepsilon$, *die zu je zwei höchstens* $(n-1)$*-dimensionale, zu je drei höchstens* $(n-2)$*-dimensionale, ... zu je* k *höchstens* $(n-k+1)$*-dimensionale, ... zu je* $n+1$ *höchstens nulldimensionale, zu je* $n+2$ (-1)*-dimensionale, d.h. leere Durchschnitte haben.* Wir drücken dieses Theorem auch kurz in folgender Weise aus: *Ein n-dimensionaler kompakter Raum kann für jedes* $\varepsilon > 0$ *kanonisch in endlich viele abgeschlossene Teilmengen mit Durchmessern* $< \varepsilon$ *zerlegt werden.*

Aus diesem allgemeinen Zerlegungsssatz folgert man mühelos, *dass zu jedem n-dimensionalen kompakten Raum A ein ihn erzeugendes kanoni-*

[1]) Diese Proceedings, **29**, S. 476.
[2]) Diese Proceedings, **29**, S. 1125.
[3]) Diese Proceedings, **30**, S. 425.
[4]) Mathem. Annalen, **98**, S. 81.

sches finites System von abgeschlossenen Mengen existiert, d. h. ein System $\{A_{m_1, m_2, \ldots m_k}\}$ von abgeschlossenen Mengen, mit folgenden Eigenschaften:

1. Für jeden endlichen Komplex von natürlichen Zahlen $m_1, m_2, \ldots m_{k-1}, m_k$ gilt $A_{m_1, m_2, \ldots m_{k-1}, m_k} \subset A_{m_1, m_2, \ldots m_{k-1}}$.

2. Zu jedem Punkt p von A existiert mindestens eine Folge natürlicher Zahlen $m_1, m_2, \ldots m_k, \ldots$, so dass p im Durchschnitt $A_{m_1} \cdot A_{m_1, m_2} \cdot A_{m_1, m_2, m_3} \ldots A_{m_1, m_2, \ldots m_k} \ldots$ enthalten ist.

3. Es gibt eine gegen Null konvergierende Folge $\{\varepsilon_k\}$ von positiven Zahlen, so dass die Durchmesser der Mengen $A_{m_1, m_2, \ldots m_k}$, welche k Indizes haben, $< \varepsilon_k$ sind [5]).

4. Für jede natürliche Zahl k existieren höchstens endlich viele nichtleere Mengen $A_{m_1, m_2, \ldots m_k}$ mit k Indizes.

Ein kurzer Ausdruck für die Bedingungen 1), 2), 3) ist, dass die abgeschlossene Mengen $A_{m_1, m_2, \ldots m_k}$ *ein den Raum A erzeugendes Mengensystem* bilden. Die Bedingung 4) besagt in dieser Ausdrucksweise, dass das erzeugende System $A_{m_1, m_2, \ldots m_k}$ *finit* ist. Dass das finite erzeugende System $A_{m_1, m_2, \ldots m_k}$ *kanonisch* sei, ist eine kurze Ausdrucksweise für die Giltigkeit der Bedingung:

5. Für jede natürliche Zahl k bilden die Mengen $A_{m_1, m_2, \ldots m_k}$ mit k Indizes eine kanonische Zerlegung von A, d. h. sie haben zur je r höchstens $(n-r+1)$-dimensionale Durchschnitte $(r = 2, 3, \ldots n+2)$.

Der Beweis des allgemeinen Einbettungssatzes beruht nun (so wie der in der ersten Mitteilung durchgeführte Beweis für die Einbettung der eindimensionalen Räume in den R_3) auf dem Fundamentallemma [6]): *Räume, die durch homologe finite Systeme von abgeschlossenen Mengen erzeugt werden, sind homöomorph.* Um zu einem vorgelegten n-dimensionalen kompakten Raum A eine homöomorphe Teilmenge des R_{2n+1} anzugeben, haben wir dem Fundamentallemma zufolge ein den Raum erzeugendes finites System und ein zu diesem System homologes System von Polytopen des R_{2n+1} zu konstruieren. Wir konstruieren, um diese verwickelte Aufgabe durchführbar zu machen, speziell ein A erzeugendes *kanonisches* finites Mengensystem und geben schrittweise ein zu diesem Mengensystem homologes Polytopensystem des R_{2n+1} an.

Ganz so wie in dem in der ersten Mitteilung erledigten Spezialfall $n = 1$ folgt die Durchführbarkeit der Konstruktion aus drei Hilfssätzen. Der *erste* behauptet, dass zu einer kanonischen Zerlegung eines n-dimensionalen Raumes ein entsprechendes Polytopensystem des R_{2n+1} unter Beobachtung von gewissen Randbedingungen angegeben werden kann. Der *zweite* behauptet, dass in einem Polytop des R_{2n+1} ein System von

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[5]) Auf Grund der Bedingung 3) kann die Bedingung 2) dahin verschärft werden, dass jeder Punkt p von A für mindestens eine Folge natürlicher Zahlen den Durchschnitt der entsprechenden Mengen bildet.

[6]) l.c. 1) S. 477.

Teilpolytopen mit vorgeschriebenem Maximaldurchmesser und eine entsprechende kanonische Zerlegung eines n-dimensionalen Raumes unter Beobachtung von gewissen Randbedingungen angegeben werden kann. Der *dritte* stellt eine Kombination der beiden ersten dar und behauptet, dass zu einem n-dimensionalen Raum eine kanonische Zerlegung in Teile *mit vorgeschriebenem Maximaldurchmesser* und *gleichzeitig* ein entsprechendes System von Polytopen des R_{2n+1} *mit vorgeschriebenem Maximaldurchmesser unter Beobachtung beiderseitiger Randbedingungen* angegeben werden kann.

Infolge der für beliebiges n ausserordentlich komplizierten Verhältnisse ist allerdings schon die Präzisierung der drei Lemmen recht langwierig.

Zunächst zwei Hilfsbegriffe: Wir bezeichnen als *n-dimensionale polyedrale Mannigfaltigkeit* eine n-stufig zusammenhängende [7]) Menge, welche Summe von endlich vielen n-dimensionalen Simplexen ist. Beispielsweise heisst eindimensionale polyedrale Mannigfaltigkeit jede zusammenhängende Menge welche Summe von endlich vielen Strecken ist. Wir bezeichnen als *n-dimensionales Polytop* eine n-dimensionale polyedrale Mannigfaltigkeit, die nicht verzweigt ist, d. h. in der es zu jedem Punkt eine Umgebung gibt, deren abgeschlossene Hülle ein n-dimensionales Simplex ist.

Wir formulieren nunmehr

Die den drei Hilfssätzen gemeinsamen Voraussetzungen: Es sei ein höchstens n-dimensionaler Raum A gegeben und eine höchstens $(n-1)$-dimensionale abgeschlossene Teilmenge B von A, welche kanonisch zerlegt ist in die Mengen $B_1, B_2, \ldots B_s$, so also, dass der Durchschnitt von je k Mengen B_1 höchstens $(n-k)$-dimensional ist $(k=2, 3, \ldots n+1)$. *Es sei ferner ein $(2n+1)$-dimensionales Polytop P gegeben und auf der Begrenzung von P ein System von $(2n-1)$-dimensionalen Polytopen $Q_1, Q_2, \ldots . Q_s$ gemäss folgenden Bedingungen:*

1. *Der Durchschnitt von k Polytopen $Q_{i_1}, Q_{i_2}, \ldots Q_{i_k}$ ist dann und nur dann leer, wenn der entsprechende Durchschnitt $B_{i_1}, B_{i_2}, \ldots B_{i_k}$ leer ist.*

2. *Wenn der Durchschnitt von k Mengen Q_i nicht leer ist, so ist er ein $(2n-2k+1)$-dimensionales Polytop.*

Es lautet dann

Hilfssatz I. Voraussetzungen: Es seien die erwähnten Voraussetzungen erfüllt. Es sei ferner A kanonisch zerlegt in die Mengen $A_1, A_2, \ldots A_r$, so dass der Durchschnitt von irgend m Mengen A_i und m' Mengen B_j höchstens $(n-m-m'+1)$-dimensional ist $(m=1, 2, \ldots n+2,\ m'=1, 2, \ldots m+1)$. [Da die Zerlegung von A in die Mengen A_i kanonisch ist, gilt die letzte Beziehung auch für $m'=0$].

Behauptung: Es existiert ein System von $(2n+1)$-dimensionalen Polytopen $P_1, P_2, \ldots P_r$, welche abgesehen von ihren etwaigen Durchschnitten mit den Mengen Q_j ganz im Innern von P enthalten sind und so, dass erstens der Durchschnitt von irgend m Mengen P_i und m'

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[7]) Diese Proceedings, **30**. S. 705.

Mengen Q_j dann und nur dann leer ist, wenn der Durchschnitt der $m+m'$ entsprechenden Mengen A_i und B_j leer ist, und dass zweitens der Durchschnitt von m Mengen P_i und m' Mengen Q_j, wenn er nicht leer ist, ein $(2n-2m-2m'+3)$-dimensionales Polytop ist, $(m=1, 2, \ldots n+2,\ m'=0, 1, \ldots n+1)$.

Der Durchschnitt von je $n+2$ unter den $r+s$ Mengen A_i und B_j ist leer. Unter den Durchschnitten von $n+1$ unter diesen $r+s$ Mengen kann es solche geben, die nicht leer sind. Jedem solchen nichtleeren Durchschnitt lassen wir einen Punkt von P entsprechen. Ist z.B. der Durchschnitt $A_{i_1} . A_{i_2} \ldots A_{i_m} . B_{j_1} . B_{j_2} \ldots B_{j_{m'}}$, $(m+m'=n+1)$ nicht leer, so lassen wir ihm einen Punkt $p^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_m}$ im Innern des Polytops $Q_{j_1} . Q_{j_2} \ldots Q_{j_{m'}}$, bzw. im Innern von P, falls $m'=0$ ist, entsprechen. Wir bezeichnen die Menge, welche ausschliesslich dem Punkt $p^{j_1, j_2 \ldots j_m}_{i_1, i_2 \ldots i_m}$ enthält, (und daher eine nulldimensionale Mannigfaltigkeit ist mit $G^{j_1, j_2 \ldots j_m}_{i_1, i_2 \ldots i_m}$. Wir haben solcherart gewisse nulldimensionale Mannigfaltigkeiten, die oben und unten insgesamt $n+1$ Indizes haben, definiert. Angenommen, es seien bereits definiert sämtliche Mengen $G^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_m}$ $(m+m'=k+1)$, welche oben und unten insgesamt $k+1$ Indizes haben, als $(n-k)$-dimensionale polyedrale Mannigfaltigkeiten. Dann definieren wir die Mengen G, welche oben und unten insgesamt k Indizes haben, durch folgende Festsetzung: Wir ordnen jedem nicht leeren Durchschnitt von m Mengen A_i und m' Mengen B_j, wo $m+m'=k$ ist, einen Punkt von P zu. Ist etwa der Durchschnitt $A_{i_1} . A_{i_2} \ldots A_{i_m} . B_{j_1} . B_{j_2} \ldots B_{j_{m'}}$, nicht leer, so ordnen wir ihm einen Punkt $p^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_m}$ zu, der im Innern des Polytops $Q_{j_1, j_2, \ldots j_{m'}}$, bzw. im Innern von P, falls $m'=0$ ist, liegt, und der ausserhalb aller bereits bestimmten Mengen G mit mehr als k Indizes gelegen ist. Wir bezeichnen sodann als $G^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_m}$ eine $(n-k+1)$-dimensionale polyedrale Mannigfaltigkeit, welche den Punkt $p^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_m}$ im Innern und auf ihrem Rand sämtliche bereits definierten Mengen $G^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_{m,i}}$ $(i=1, 2, \ldots r)$ und $G^{j_1, j_2 \ldots j_{m'}, j}_{i_1, i_2 \ldots i_m}$ $(j=1, 2, \ldots r)$ enthält. Dabei sorgen wir dafür, dass alle Punkte von $G^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_m}$ abgesehen von jenen, welche in Mengen $G^{j_1, j_2 \ldots j_{m'}}_{i_1, i_2 \ldots i_{m,i}}$ und $G^{j_1, j_2 \ldots j_{m'}, j}_{i_1, i_2 \ldots i_m}$ liegen, ganz im Innern von $Q_{j_1} . Q_{j_2}, \ldots Q_{j_{m'}}$, bzw. von P, falls $m'=0$ ist, liegen. Dabei kann diese Zuordnung von $(n-k+1)$-dimensionalen polyedralen Mannigfaltigkeiten G mit oben und unten insgesamt k Indizes zu den nicht leeren Durchschnitten von $m+m'=k$ Mengen A_i und B_j *derart* vorgenommen werden, (und dies ist der springende Punkt!) dass je p von diesen Mengen G mit k Indizes höchstens Punkte einer Mannigfaltigkeit G mit insgesamt mehr als k Indizes gemein haben und zwar Punkte von jener Mannigfaltigkeit, welche unten und oben alle jene Indizes besitzt, die in den p Mannigfaltigkeiten unten, bzw. oben auftreten. *Diese Bedingung ist erfüllbar, da es sich* für $k>1$ um $(n-k+1)$-dimensionale, also *um höchstens n-dimensionale polyedrale Mannigfaltigkeiten eines R_{2n+1} handelt* und da im R_{2n+1} zwei n-dimensionale

Ebenen "im allgemeinen" fremd sind. Sollten also bei der Bestimmung der höchstens n-dimensionalen Mannigfaltigkeiten G unerwünschte Schnittpunkte auftreten, so können dieselben durch leichte Modifikation der Mannigfaltigkeiten weggeschafft werden. Im Fall $n = 1$ sind die Mannigfaltigkeiten mit einem Index eindimensionale polyedrale Mannigfaltigkeiten des R_3 und wenn solche Mannigfaltigkeiten "zufällig" unerwünschte Schnittpunkte haben, können dieselben durch leichte Modifikation der Mannigfaltigkeiten (indem man die sich schneidenden Strecken durch windschiefe Streckenzüge ersetzt), weggeschafft werden.

Wir konstruieren also in der angegebenen Art ausgehend von den Mengen G, die oben und unten insgesamt $n+1$ Indizes haben, schrittweise die Mengen G mit weniger Indizes, bis wir endlich zu den n-dimensionalen polyedralen Mannigfaltigkeiten $G_1, G_2, \ldots G_r$ gelangen, welche den Mengen $A_1, A_2, \ldots A_r$ entsprechen.

Die Herleitung von $(2n+1)$-dimensionalen Polytopen $P_1, P_2, \ldots P_r$ mit den in Hilfssatz I geforderten Eigenschaften aus den so konstruierten Mannigfaltigkeiten $G_1, G_2, \ldots G_r$ ist eine einfache elementargeometrische Aufgabe. Man ersetzt jede nulldimensionale Mannigfaltigkeit G, die oben und unten insgesamt $n+1$ Indizes hat, durch ein eindimensionales Polytop (d.h. eine Strecke, welches die nulldimensionale Mannigfaltigkeit enthält, wobei die Strecken paarweise fremd und so klein gewählt werden können, dass jede von ihnen im Innern derjenigen Polytope $Q_{j_1} \cdot Q_{j_2} \ldots Q_{j_{m'}}$ liegt, in deren Innern die entsprechende nulldimensionale Mannigfaltigkeit G liegt). Man ersetzt sodann jede eindimensionale Mannigfaltigkeit G, die oben und unten insgesamt n Indizes hat, durch ein dreidimensionales Polyeder mit n Indizes, welches jene Strecken P mit $n+1$ Indizes auf seiner Begrenzung enthält, für welche die entsprechenden Mengen G mit $n+1$ Indizes in der entsprechenden Mannigfaltigkeit mit n Indizes enthalten sind, wobei das Polyeder überdies so klein gewählt wird, dass es abgesehen von diesen Strecken auf seiner Begrenzung ganz im Innern jener Polytope $Q_{j_1} \cdot Q_{j_2} \ldots Q_{j_{m'}}$ liegt, in deren Innerem die entsprechende Mannigfaltigkeit G mit n Indizes angenommen wurde. Da die Mengen G mit n Indizes zu je dreien fremd sind und zu je zweien höchstens Punkte G mit $n+1$ Indizes gemein haben, so können auch die P mit n Indizes so bestimmt werden, dass sie zu je dreien fremd sind und zu je zwei höchstens eine Strecke P mit $n+1$ Indizes gemein haben. Fährt man in dieser Weise fort, so genügen die $(2n+1)$-dimensionalen Polytope P_t mit einem Index, zu denen man schliesslich gelangt, offenbar den Forderungen des Hilfssatzes I.

Hilfssatz II. Es seien die gemeinsamen Voraussetzungen erfüllt und eine Zahl $\eta > 0$ gegeben. Behauptung: Es existiert eine kanonische Zerlegung von A in gewisse abgeschlossene Teilmengen $A_1, A_2, \ldots A_r$ und ein System von $(2n+1)$-dimensionalen Polytopen $P_1, P_2, \ldots P_r$ mit Durchmessern $< \eta$, so dass die folgenden Bedingungen erfüllt sind:

1. *Der Durchschnitt von irgend m Polytopen P_t und m' Polytopen*

Q_j ist dann und nur dann leer, wenn der Durchschnitt der $m+m'$ entsprechenden Mengen A_i und B_j leer ist.

2. *Ist die Menge $P_{i_1} \cdot P_{i_2} \ldots P_{i_m} \cdot Q_{j_1} \cdot Q_{j_2} \ldots Q_{j_{m'}}$, nicht leer, so ist sie ein $(2n-2m-2m'+3)$-dimensionales Polytop.*

Für $n=0$ ist die Behauptung trivial. In diesem Fall ist A nulldimensional, die Mengen B_j sind leer, P ist eine Strecke, die Mengen Q_j sind leer. Wir haben, um die Forderungen von Hilfssatz 2 zu erfüllen, P bloss durch eine Teilstrecke mit einem Durchmesser $<\eta$ zu ersetzen.

Der Fall $n=1$ ist bereits in der ersten Mitteilung erledigt worden, jedoch mit Hilfe eines Verfahrens der Bildung linearer Ketten, welches nicht ohne weiteres auf höhere Dimensionen verallgemeinerbar ist. Wir führen daher hier einen Beweis dieses Falles durch, welcher sich auf beliebige höhere Dimensionen übertragen lässt. Es ist im Fall $n=1$ die Menge A höchstens eindimensional, die s abgeschlossenen Mengen B_j sind paarweise fremd und nulldimensional, P ist ein dreidimensionales Polyeder, die s Mengen Q_j sind paarweise fremde Teilstrecken der Begrenzung von P. Hilfssatz 2 behauptet: Es existiert *erstens* eine Zerlegung von A in endlich viele abgeschlossene Mengen $A_1, \ldots A_r$, so dass die Durchschnitte von je zwei Mengen A_i höchstens nulldimensional, die Durchschnitte von je drei Mengen A_i und die Durchschnitte von zwei Mengen A_j und einer Menge B_j leer sind, und es existiert *zweitens* ein System von r Polyedern $P_1, \ldots P_r$ mit Durchmessern $<\eta$, so dass je zwei P_i, bzw. eine Menge P_i und eine Menge Q_j dann und nur dann einen nichtleeren Durchschnitt u.zw. eine Strecke als Durchschnitt haben, wenn die beiden entsprechenden Mengen A_i, bzw. die entsprechenden Mengen A_i und B_j einen nichtleeren Durchschnitt haben, und dass der Durchschnitt von je drei Mengen P_i und von je zwei Mengen P_i und einer Menge Q_j leer ist.

Zum Beweise bemerken wir zunächst, dass es möglich ist, P in zwei Teilpolyeder P^1, P^2, jedes derselben wieder in zwei Teilpolyeder $P^{11}, P^{12}, P^{21}, P^{22}$ zu zerlegen u.s.w. derart, dass man nach einer endlichen Anzahl von Schritten, etwa nach m Schritten, 2^m Teilpolyeder $P^{i_1, i_2 \ldots i_m}$ $(i_k = 1,2)$ erhält, deren Durchmesser $<\eta$ sind. Wir betrachten die beiden ersten Polyeder P^1 und P^2. Ist eines von ihnen, etwa P^2, zur Menge $\sum_{j=1}^{s} Q_j$ fremd, so tilgen wir P^2 und ordnen P^1 der Menge A zu. Liegt dieser Fall nicht vor, so sind die Mengen Q_j teils in P^1, teils in P^2 enthalten. Es möge etwa P^1 die Mengen $Q_{i_1}, Q_{i_2}, \ldots Q_{i_{m_1}}$ enthalten, es möge P^2 die Mengen $Q_{j_1}, Q_{j_2}, \ldots Q_{j_{m_2}}$ enthalten, und es mögen die Mengen $Q_{k_1}, Q_{k_2}, \ldots Q_{k_{m_3}}$ sowohl Punkte mit P^1 als auch Punkte mit P^2 gemein haben. Wir betrachten zunächst diese letzteren Mengen Q_{k_i} und die ihnen entsprechenden Mengen $B_{k_1}, B_{k_2}, \ldots B_{k_{m_3}}$. Jede dieser Mengen B_{k_i} zerlegen wir in zwei zueinander fremde abgeschlossene Teilmengen $B^1_{k_i}$ und $B^2_{k_i}$. Sollte eine solche Zerlegung nur so möglich sein, dass einer der beiden Summanden leer ausfällt (m.a.W. wenn die Menge B_{k_i} nur einen

einzigen Punkt enthält), dann wählen wir $B^2_{k_i}$ als leer und verkleinern das Polytop P^2 ein wenig in der unmittelbaren Nachbarschaft der Strecke, welche P^2 mit B_{k_i} gemein hat, so dass das modifizierte Polyeder zu Q_{k_i} fremd ist. Wenn eine Zerlegung von B_{k_i} in zwei fremde nichtleere abgeschlossene Teilmengen $B^1_{k_i}$ und $B^2_{k_i}$ möglich ist, so nehmen wir sie vor. Wir lassen dann diese beiden Mengen zwei fremden Teilstrecken $Q^1_{k_i}$ $Q^2_{k_i}$ von Q_{k_i} entsprechen und verkleinern P^1 und P^2 ganz wenig in der Umgebung von Q_{k_i}, so dass die Begrenzung der modifizierten Menge P^1 mit der Strecke Q_{k_i} bloss die Teilstrecke $Q^1_{k_i}$ und dass die Begrenzung der modifizierten Menge P^2 mit der Strecke Q_{k_i} bloss die Teilstrecke $Q^2_{k_i}$ gemein hat. Auf diese Weise behandeln wir der Reihe nach alle Mengen Q_{k_i}, die sowohl mit P^1 als auch mit P^2 Punkte gemein haben. Sodann bilden wir eine Menge A^1, welche die abgeschlossene Hülle ist von einer offenen Menge mit höchstens nulldimensionaler Begrenzung, welche offene Menge die Mengen $B_{i_1}, B_{i_2}, \dots B_{i_{m_1}}, B^1_{k_1}, B^1_{k_2}, \dots B^1_{k_{m_3}}$ enthält, und so dass die abgeschlossene Mengen A^1 zu den Mengen $B_{j_1}, B_{j_2}, \dots B_{j_{m_2}}$, $B^2_{k_1}, B^2_{k_2}, \dots B^2_{k_{m_3}}$ fremd ist. Die *Existenz* einer derartigen Menge A^1 ergibt sich in ganz einfacher Weise durch Anwendung des Borelschen Theorems. Wir ordnen die erwähnte Menge A^1 dem Polyeder P^1 zu und ordnen die Menge $\overline{A - A^1} = A^2$ dem Polyeder P^2 zu. Haben A^1 und A^2 einen leeren Durchschnitt, so modifizieren wir die Polyeder P^1 und P^2 in der Nähe der ihnen gemeinsamen Begrenzungsfläche, indem wir die Polyeder etwas verkleinern, so dass sie *fremd* werden. Haben A^1 und A^2 einen nichtleeren Durchschnitt, dann ist derselbe, da A^1 die abgeschlossene Hülle einer offenen Menge mit nulldimensionaler Begrenzung ist, nulldimensional. In diesem Fall modifizieren wir P^1 und P^2 in der Nähe der ihnen gemeinsamen Begrenzungsfläche, indem wir die Polyeder etwas verkleinern, derart, dass sie nach der Modifikation bloss eine *Strecke* gemeinsam haben, die zu sämtlichen Strecken $Q_1, Q_2, \dots Q_s$ fremd ist. Diese Strecke $P^1 \,.\, P^2$ ordnen wir der Menge $A^1 \,.\, A^2$ zu.

Nun sehen wir: Für jedes der beiden modifizierten Polyeder P^1 und P^2 sind dieselben Voraussetzungen erfüllt, wie eingangs für P. Bezeichnen wir mit P^{11} und P^{12} den Durchschnitt der eingangs erwähnten Polyeder P^{11} und P^{12} mit dem modifizierten Polyeder P^1, so liegen auf der Begrenzung dieses letzteren paarweise fremde Strecken, nämlich $Q_{i_1}, Q_{i_2}, \dots$ $Q_{i_{m_1}}, Q^1_{k_1}, Q^2_{k_2}, \dots Q^1_{k_{m_3}}, P^1 \,.\, P^2$, vor und diesen Strecken entsprechen die paarweise fremden nulldimensionalen abgeschlossenen Teilmengen B_{i_1}, $B_{i_2}, \dots B_{i_{m_1}}, B^1_{k_1}, B^1_{k_2}, \dots B^1_{k_{m_3}}, A_1 \,.\, A_2$ der Menge A^1. Wir können daher nach dem eben durchgeführten Verfahren den Polyedern P^{11} und P^{12} zwei Mengen A^{11} und A^{12} entsprechen lassen, ebenso Mengen A^{21} und A^{22} definieren und erhalten so durch wiederholte Anwendung des Schlusses ein System von Mengen $A^{k_1 k_2 \dots k_n}$, $(k_i = 1, 2)$, welches den

Polyedern $P^{k_1 k_2 \cdots k_n}$ entspricht und daher den Forderungen von Hilfssatz 2 genügt. Damit ist der Fall $n = 1$ erledigt.

Wie dieses Verfahren auf höhere Dimensionen zu übertragen ist, ist klar. Wir wollen zunächst noch den Fall $n = 2$ durchführen. Es ist dann die Menge A höchstens zweidimensional, die s Mengen B_j sind höchstens eindimensionale Mengen, die zu je zwei höchstens nulldimensionale und zu je drei leere Durchschnitte haben. P ist ein fünfdimensionales Polytop, auf dessen Begrenzung s dreidimensionale Polyeder gegeben sind, die zu je zwei dann und nur dann einen nicht leeren Durchschnitt u.zw. eine Strecke als Durchschnitt haben, wenn die entsprechenden Mengen B_j einen nicht leeren Durchschnitt haben, und die zu je dritt fremd sind. Behauptet wird *erstens* die Existenz eines Systems von abgeschlossenen Mengen $A_1, A_2, \ldots A_r$, so dass der Durchschnitt von je m Mengen A_i und m' Mengen B_j höchstens $(2 - m - m' + 1)$-dimensional, also höchstens $(3-m-m')$-dimensional ist $(m = 2, 3, 4 \,.\, m' = 0, 1, 2, 3)$ und *zweitens* die Existenz eines Systems von fünfdimensionalen Polytopen $P_1, P_2, \ldots P_r$, so dass der Durchschnitt von m Mengen P_i und m' Mengen Q_j dann und nur dann nicht leer u.zw. ein $(7 - 2m - 2m')$-dimensionales Polyeder ist, falls der Durchschnitt der $m + m'$ entsprechenden Mengen A_i und B_j nicht leer ist.

Wir bilden so wie im Fall $n = 1$ durch sukzessive Zweiteilungen ein System von fünfdimensionalen Polytopen $P^{k_1 k_2 \cdots k_n}$ $(k_i = 1, 2)$, so dass die Polytope eines gewissen m-ten Schrittes sämtlich Durchmesser $< \eta$ haben. Wir betrachten sodann die Polytope P^1 und P^2 des ersten Schrittes. Ist eines von ihnen, etwa P^2, zu $\sum_{j=1}^{s} Q_j$ fremd, dann tilgen wir es und ordnen P^1 der Menge A zu. Andernfalls hat sowohl P^1 als auch P^2 Punkte mit $\sum_{j=1}^{s} Q_j$ gemein. Jedes der dreidimensionalen Polyeder Q_j ist entweder in einem der beiden Polytope P^1, P^2 enthalten, oder es hat Punkte mit beiden gemein. Es mögen etwa die Polyeder $Q_{i_1}, Q_{i_2}, \ldots Q_{i_{m_1}}$ in P^1 enthalten sein, die Polyeder $Q_{j_1}, Q_{j_2}, \ldots Q_{j_{m_2}}$ in P^2, und es mögen die Polyeder $Q_{k_1}, Q_{k_2}, \ldots Q_{k_{m_3}}$ sowohl mit P^1 als auch mit P^2 Punkte gemein haben. In den Polyedern Q_k der letzteren Art wird durch die Zerlegung $Q_k = Q_k \,.\, P^1 + Q_k \,.\, P^2$ eine Zerlegung in zwei Teilpolyeder induziert, welche alle Voraussetzungen des bereits erledigten Falles $n = 1$ erfüllt. Es ist ja Q_k ein dreidimensionales Polyeder, das auf seiner Begrenzung paarweise fremde Strecken enthält, nämlich die Mengen $Q_k \,.\, Q_l$, $(l = 1, 2, \ldots s)$ soweit sie nicht leer sind. Dem Polyeder Q_k entspricht die höchstens eindimensionale Menge B_k mit endlich vielen paarweise fremden abgeschlossenen nulldimensionalen Teilmengen, nämlich den Mengen $B_k \,.\, B_l$ $(l = 1, 2, \ldots s)$, soweit sie nicht leer sind. Es können daher zufolge der für $n = 1$ bewiesenen Konstruktion durch leichte Modifikation der Polyeder $P^1 \,.\, Q_k$ und $P^2 \,.\, Q_k$ zwei Polyeder Q_k^1 und Q_k^2 konstruiert und eine ihnen entsprechende Zerlegung von B_k in Teil-

mengen B_k^1 und B_k^2 angegeben werden, so dass der Durchschnitt $B_k^1 . B_k^2$ höchstens nulldimensional und zu den Mengen $B_k . B_l$ fremd ist und dass der Durchschnitt $Q_k^1 . Q_k^2$ leer ist, falls $B_k^1 . B_k^2$ leer ist, und eine zu den Strecken $Q_i . Q_j$ fremde Strecke ist, falls $Q_k^1 . Q_k^2$ nicht leer ist. Auf diese Weise behandeln wir alle Polyeder $Q_{k_1}, Q_{k_2}, \ldots Q_{k_{m_3}}$. Sodann bilden wir eine Menge A^1, welche die abgeschlossene Hülle ist von einer offenen Menge O^1 mit einer höchstens eindimensionalen Begrenzung derart, dass *erstens* O^1 die Mengen $B_{i_1}, B_{i_2}, \ldots B_{i_{m_1}}$ und die Mengen $B_{k_1}^1, B_{k_2}^1, \ldots B_{k_{m_3}}^1$ (ausgenommen ihre Durchschnitte mit den Mengen $B_{k_j}^2$) enthält, dass *zweitens* die Begrenzung von O^1 die Mengen $B_{k_1}^1 . B_{k_1}^2, B_{k_2}^1 . B_{k_2}^2, \ldots B_{k_{m_3}}^1 . B_{k_{m_3}}^2$ enthält und dass *drittens* die Menge A^1 zu den Mengen $B_{j_1}, B_{j_2}, \ldots B_{j_{m_2}}$, und zu den Mengen $B_{k_1}^2, B_{k_2}^2, \ldots B_{k_{m_3}}^2$ (ausgenommen ihre Durchschnitte mit den Mengen $B_{k_j}^1$) fremd ist. Wir lassen diese Menge, deren Existenz wieder durch einfache Anwendung des Borelschen Theorems erwiesen werden kann, dem Polytop P^1 entsprechen und lassen die Menge $A_2 = \overline{A - A^1}$ dem Polytop A^2 entsprechen. Der Durchschnitt $A^1 . A^2$ ist die Begrenzung von A^1, also höchstens eindimensional. Ist diese Menge leer, so modifizieren wir P^1 und P^2, indem wir sie in der Nähe ihrer vierdimensionalen Begrenzungsebenen etwas verkleinern, so dass sie zu einander fremd werden. Andernfalls verkleinern wir P^1 und P^2 in der Nähe ihrer vierdimensionalen Begrenzungsebene, indem wir sie etwas verkleinern, so dass sie bloss ein dreidimensionales Polyeder mit einander gemein haben, welches die paarweise fremden Strecken $Q_{k_1}^1 . Q_{k_1}^2, \ldots Q_{k_{m_3}}^1 . Q_{k_{m_3}}^2$ auf seiner Begrenzung enthält.

Dasselbe Verfahren wenden wir nun auf die modifizierten Polytope P^1 und P^2 bzw. auf die Mengen A^1 und A^2 an, bis wir endlich zu modifizierten Polytopen $P^{n_1 n_2 \cdots n_m}$ und zu entsprechenden Mengen $A^{n_1 n_2 \cdots n_m}$ gelangen, welche den Forderungen von Hilfssatz 2 genügen.

Diese Konstruktion kann offenbar, sobald sie für $n-1$ durchgeführt ist, für n durchgeführt werden. Wir erzeugen aus dem $(2n+1)$-dimensionalen Polytop P durch sukzessive Zweiteilungen Polytope $P^{n_1 n_2 \cdots n_m}$ mit Durchmessern $< \eta$. Wir betrachten dann zuerst die Polytope P^1 und P^2. Von den $(2n-1)$-dimensionalen Polytopen Q_j auf der Begrenzung von P sind einige, etwa $Q_{i_1}, Q_{i_2}, \ldots Q_{i_{m_1}}$, ganz in P^1 enthalten, einige, etwa $Q_{j_1}, Q_{j_2}, \ldots Q_{j_{m_2}}$, sind ganz in P^2 enthalten, einige, etwa $Q_{k_1}, Q_{k_2}, \ldots Q_{k_{m_3}}$, haben sowohl mit P^1 als auch mit P^2 Punkte gemein. In den letzteren Polytopen wird durch die Zerlegung $P = P^1 + P^2$ eine Zerlegung induziert, welche wir auf Grund unserer induktiven Annahme bereits beherrschen. Wir können die Menge B_{k_i} in zwei Teilmengen $B_{k_i}^1$ und $B_{k_i}^2$ zerlegen und entsprechend zwei $(2n-1)$-dimensionale Polytope $Q_{k_i}^1$ und $Q_{k_i}^2$ bestimmen, deren Durchschnitt leer ist, falls $B_{k_i}^1 . B_{k_i}^2$ leer ist, und deren Durchschnitt ein $(2n-3)$-dimensionales Polytop ist, falls $B_{k_i}^1 . B_{k_i}^2$ nicht leer ist. Wir können sodann zwei abgeschlossene Mengen A^1 und

A^2 bilden, $A^2 = \overline{A - A^1}$, und sie den Polytopen P^1 und P^2 entsprechen lassen, wobei A^1 die abgeschlossene Hülle einer offenen Menge O^1 mit höchstens $(n-1)$-dimensionaler Begrenzung ist, so dass *erstens* O^1 alle Mengen $B_{i_1}, B_{i_2}, \ldots B_{i_{m_1}}$ und die Mengen $B^1_{k_1}, B^1_{k_2}, \ldots B^1_{k_{m_3}}$ (abgesehen von ihren Durchschnitten mit den Mengen B^2_k) enthält, dass *zweitens* die Begrenzung von O^1 die Mengen $B^1_{k_i} . B^2_{k_i}$ enthält, und dass *drittens* die Menge A^1 zu den Mengen $B_{j_1}, B_{j_2}, \ldots B_{j_{m_2}}$ und zu den Mengen $B^2_{k_1}, B^2_{k_2}, \ldots B^2_{k_{m_3}}$ (abgesehen von ihren Durchschnitten mit den Mengen $B^1_{k_i}$) fremd ist. Je nachdem der Durchschnitt $A^1 . A^2$ leer oder nicht leer ist, modifizieren wir P^1 und P^2 in der Nähe ihrer $2n$-dimensionalen Begrenzungsfläche, indem wir die beiden Polytope etwas verkleinern, so dass sie zu einander fremd werden, bzw. ein $(2n-1)$-dimensionales Polytop als Durchschnitt haben, welches auf seiner Begrenzung die $(2n-3)$-dimensionalen Polytope $B^1_{k_i} . B^2_{k_i}$ enthält. Die Wiederholung dieses Schlusses führt zu Mengen $A^{n_1 n_2 \cdots n_m}$, welche den modifizierten Polytopen $P^{n_1 n_2 \cdots n_m}$ entsprechen und die Forderungen von Hilfssatz 2 erfüllen.

Hilfssatz 3 ergibt sich durch Kombination der Hilfssätze 1 und 2 ganz so, wie dies für den Fall $n = 1$ in der ersten Mitteilung auseinandergesetzt wurde:

Hilfssatz III. Es seien die gemeinsamen Voraussetzungen erfüllt und zwei Zahlen $\varepsilon > 0$ und $\eta > 0$ gegeben. Behauptung: Es existiert ein System $A_1, A_2, \ldots A_r$ von abgeschlossenen Mengen mit Durchmessern $< \varepsilon$, welches die Voraussetzungen von Hilfssatz 1 erfüllt, und ein System von Polytopen $P_1, P_2, \ldots P_r$ mit Durchmessern $< \eta$, welche der Behauptung von Hilfssatz 1 genügen.

Zum Beweise geben wir zunächst, was auf Grund des allgemeinen Zerlegungstheorems möglich ist, ein System von Mengen $A'_1, A'_2, \ldots A'_p$ mit Durchmessern $< \varepsilon$ an, welche den Voraussetzungen von Halfssatz 1 genügen, und bestimmen sodann nach Hilfssatz 1 ein diesen Mengen entsprechendes System von Polytopen $P'_1, P'_2, \ldots P'_p$. Wenn in diesem Polytopensystem sämtliche Polytope Durchmesser $< \eta$ haben, so sind wir am Ziel. Andernfalls wenden wir auf jene Polytope, welche Durchmesser $\geqq \eta$ haben, der Reihe nach Hilfssatz 2 an, bestimmen für jedes dieser Polytope P'_i ein System von Polytopen P^i_k mit Durchmessern $< \eta$ und ein entsprechendes System von abgeschlossenen Mengen A^i_k, die als Teilmengen von A'_i sicherlich Durchmesser $< \varepsilon$ haben. Das System aller so bestimmten Polytope P^i_k und der entsprechenden Mengen A^i_k erfüllt, wenn wir sie mit $P_1, P_2, \ldots P_r$, bzw. mit $A_1, A_2, \ldots A_r$ bezeichnen, die Forderungen von Hilfssatz 3.

Ist ein n-dimensionaler kompakter Raum A gegeben, so kann man auf Grund von Hilfssatz 3 ein ihn erzeugendes finites System von abgeschlossenen Mengen und schrittweise ein dazu homologes System von $(2n+1)$-dimensionalen Polytopen des R_{2n+1} konstruieren, ganz so, wie

dies für den Fall $n = 1$ in der ersten Mitteilung durchgeführt wurde, womit der Einbettungssatz bewiesen ist.

Bezüglich der Details dieses schwierigen Beweises muss auf eine demnächst erscheinende ausführliche Darstellung verwiesen werden. Erwähnt werde hier nur das (von mir schon an anderer Stelle formulierte) Problem, ob nicht für gewisse natürliche Zahlen n alle n-dimensionalen Räume schon in einen weniger als $(2n + 1)$-dimensionalen Cartesischen Raum einbettbar sind. Ich halte diese Behauptung, die für $n = 1$ offenbar unrichtig ist, für *nicht* wahrscheinlich, vermute vielmehr, *dass für jedes n und für jedes* $m \leqq 2n + 1$ *ein n-dimensionaler Raum existiert, welcher in den* R_m *aber nicht in den* R_{m-1} *einbettbar ist.* Sichergestellt ist freilich nicht einmal die Existenz von Flächen, welche nicht in den R_4 einbettbar sind. Möglicherweise ist schon das Flächengerüst, welches entsteht, wenn ein fünf-dimensionaler Würfel durch 9-Teilung seiner Kanten in 9^5 Teilwürfel geteilt wird, nicht in den R_4 einbettbar; ja es ist denkbar, dass selbst das Flächengerüst, welches entsteht, wenn ein fünfdimensionaler Würfel durch Drei-, bzw. Zweiteilung seiner Kanten in 3^5, bzw. 2^5 Teilwürfel geteilt wird, nicht in den R_4 einbettbar ist.

Physics. — *On the increase of the sparking potential of a gas mixture by irradiation.* By F. M. PENNING. (Communicated by Dr. G. HOLST.)

(Communicated at the meeting of March 31, 1928).

Some time ago measurements were published elsewhere [1]) on the sparking potential of neon and argon to which very small amounts of other gases had been added. It was shown that in general for large values of *pd* (pressure $\times$ distance between the electrodes) the sparking potential was much lowered if the ionisation potential of the admixed gas was smaller than the excitation potential of the main gas. For the explanation of this phenomenon it was assumed that metastable atoms of the main gas ionise the atoms of the admixture [2]).

An unambiguous proof for the correctness of this explanation was found in the influence of radiation on the sparking potential of such a gas mixture. This will be shown in what follows. When neon is used as the main gas, metastable atoms in the $2s_3$- and $2s_5$-state will be formed. In case an admixture of for instance 0.001 % argon is present, these metastable atoms with their long life time [3]) will have a considerable chance of ionising argon atoms. If, however, the life time of the metastable atoms is shortened artificially, the probability of the ionisation of an argon atom becomes smaller and the sparking potential increases. This shortening of the life time can be brought about by irradiation with neon light, for instance that of a positive column. Such a column emits for a large amount spectral lines of the type $2s_5$—$2p$ and $2s_3$—$2p$ [4]); by the light of these lines metastable atoms in the $2s_5$- and $2s_3$-states may be brought into one of the not metastable $2p$-states. It is true that for the greater part they will fall back from this state into the $2s_5$-state [5]), but a considerable number will also return, by way of the $2s_2$- and the $2s_4$-state, into the normal state. This latter number will not be able to ionise argon atoms; so, in order to obtain the wanted number of positive ions the potential on the tube must be made higher. In other words, illumination by the light of a neon tube will increase the sparking potential of the gasmixture.

In fig. 1 these processes are shown schematically. For the sake of

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[1]) F. M. PENNING, Naturwissenschaften **15**, 818, 1927; Zts. f. Phys. **46**, 335, 1928.

[2]) In the meantime this ionisation by metastable atoms has been demonstrated also optically, see R. FRERICHS, Ann. d. Phys. **85**, 362, 1928.

[3]) H. B. DORGELO and T. P. K. WASHINGTON, These Proceedings **30**, 33, 1927.

[4]) H. B. DORGELO, Physica **5**, 90, 1925.

[5]) H. B. DORGELO and W. DE GROOT, Zts. f. Physik **36**, 897,1926.

clearness only the two most important $2s$-states ($2s_4$- and the metastable $2s_5$-state) are given, while the tenfold $2p$-state is represented by one line. Transitions which are not accompanied by radiation are shown by dotted lines; besides the transition from $2s_5$ to $2s_4$ (which limits probably in pure neon the life time of the $2s_5$-atoms) we have in the case of an argon admixture the transition from $2s_5$ to the normal state by the way of the ionisation of an argon atom. Fig 1*b* and 1*a* show the transitions in case irradiation when a neon column is applied (fig. 1*b*) or not (fig. 1*a*).

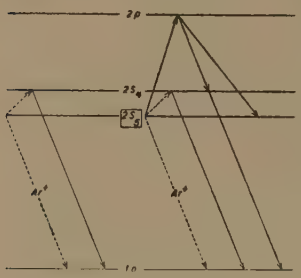

Fig. 1*a*. Fig. 1*b*.

Indeed, this effect of irradiation could be demonstrated. As an example the following experiment may be mentioned.

A discharge tube contained 20 mm neon with about 0.001 % argon, the electrodes were plane, parallel iron plates of $2\frac{1}{2}$ cm diameter, the distance between them was 1 cm. The discharge space was surrounded by a positive column tube, filled with 10 mm neon. This tube had been bent into a circle of about 3 cm internal diameter. A current of 15 mA through this latter tube gave rise to an increase of the sparking potential of 28 V; for 100 mA the increase was 50 V [1]).

It should be remarked that a considerable increase of the sparking potential can be obtained only if not more than a very small amount of admixture is present. Indeed, with a large amount of admixture a metastable atom will ionise already after a small fraction of its natural lifetime. As a consequence of this the probability that a metastable atom is reduced to the normal state by irradation becomes much less. So a discharge tube of the type already described, but filled with 25 mm neon and a drop of mercury (at room temperature 0.004 % *Hg*) showed no increase of the sparking potential when it was exposed to irradiation in the way as mentioned above. If however the vapour pressure of the mercury was lowered by cooling, then the effect showed itself again.

In the second place the pressure of the main gas should not be too high. As was stated higher up the destruction of the metastable atoms by irradiation is caused by the transition of the atoms from the $2p$-states via the $2s_2$- and $2s_4$-state to the normal state. If the pressure becomes too high, the time t_1 between two collisions with gasatoms will become smaller than the natural life time t_2 of the s_2- and s_4-states (for neon at a pressure of 20 mm t_1 is already [2]) 10^{-8} sec., while t_2 is about 10^{-8} or 10^{-7} sec. [3]).) Now at a collision an atom in the s_2- or s_4-state has a considerable chance

[1]) See for these experiments also Physica **8**, 137, 1928.

[2]) For a normal neon atom t_1 is about 10^{-8} sec., therefore for an excited atom probably considerably lower.

[3]) Cf. the experimental values of W. WIEN: Ann. d. Phys. **73**, 483, 1924.

to pass into a metastable state of lower energy [1]). It is clear that this effect diminishes the influence of the irradiation. The experiments point into the same direction. In the first efforts to detect the effect, the electrodes were very small and very near to each other (about 1 mm). In this way we were able to obtain a large radiation density by concentrating the light of the neon tube between the electrodes with a lense. In order to have appropriate values of *pd*, the pressure had to be made much higher, so it was taken 600 mm. Although in this experiment the decrease of the sparking potential by small amounts of argon showed itself in the usual way, an intense irradiation had no effect, in agreement with the remarks made above.

Natuurkundig Laboratorium der
N.V. PHILIPS' *Gloeilampenfabrieken.*

Eindhoven, 28 Maart 1928.

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[1]) Cf. J. FRANCK and P. JORDAN, Anregung von Quantensprüngen durch Stösse, Berlin 1926, p. 232 etc., P. D. FOOTE, Phys. Rev. **30**, 288, 1927; W. ORTHMANN and P. PRINGSHEIM, Zts. f. Phys. **46**, 160, 1927

Chemistry. — *The influence of the solvent on the optical rotation dispersion.* By A. L. TH. MOESVELD. (Communicated by Prof. ERNST COHEN.)

(Communicated at the meeting of October 27, 1928).

1. In a communication "on the influence of pressure on the reaction velocity and the function of the medium" A. L. TH. MOESVELD and WILHELMA A. T. DE MEESTER [1]) have directed attention to a statement of LANDOLT's [2]) concerning the influence of solvents on the rotation of dissolved active compounds, and the latter has enlarged on this subject in her thesis [3]). The conception about the mutual influence of solvent and dissolved substance, given in the two papers just mentioned, enabled us to explain well-known facts about the function of pressure during the process of chemical reactions, while new inferences could be confirmed by experiments. In this conception the medium has no longer exactly the same composition throughout; the composition of the environment of a molecule of the dissolved substance differs from the gross composition of the solvent. At the same time a deformation appears in the molecule of the dissolved substance. The conception of LANDOLT about the function of the solvent with respect to the rotation of a dissolved optically active substance appears best from his own words:

"Erscheinungen solcher Art (the change of the rotation by dissolving in another solvent) lassen sich vielleicht durch die Vorstellung erklären, dass, wenn zwischen die Moleküle einer aktiven Substanz (Terpentinöl) andere Moleküle (Alkohol) treten, dadurch eine gewisse Modification in der Struktur der ersteren hervorgebracht wird, und zwar in der Weise, dass in jedem Molekül der gegenseitige Abstand der Atome, ihre Anordnung im Raume, sowie die Art der Atombewegungen, sich etwas ändert. Dies wird in um so stärkerem Grade der Fall sein, je grösser die Zahl der inaktiven Teilchen ist."

In greater support of the views developed, it seemed to us of special importance to trace in how far changes of rotation occur, when bornyl-acetate to different concentrations is dissolved in mixtures of water and alcohol of different composition. Various reasons made it impossible to complete this investigation in this direction, the more so as from preliminary

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[1]) Verslagen der Kon. Akad. v. Wet. Amsterdam, **36**, 827 (1927), see p. 831, note 1. Proc. **30**. 1039 (1928) esp. p. 1043, footnote 1.

[2]) Das optische Drehungsvermögen. Braunschweig 1898, p. 210.

[3]) The influence of pressure on the reaction velocity and the function of the medium. Thesis, Utrecht, 1928, specially chapter X, p. 115.

investigations appeared that the accuracy would have to be considerably raised in order to draw definite conclusions. The investigation described hereafter, which is on a more or less modified plan, has given results which are in complete agreement with the views mentioned above.

2. This modification concerns what follows: Though, of course the exposition in the last paragraph on page 116 of the thesis mentioned above, about the advantages of the study of a physical constant which refers to only one of the components of the solution, holds good in every respect. yet the choice of the optical rotation for a definite colour is somewhat arbitrary. When for a definite colour the rotation changes from solvent to solvent, it may also be expected that the dispersion will every time have a different value. That is to say that for every colour another relation of the optical rotations for two different solvents will be found, so that the optical rotation alone can never be a measure for a contingent deformation. It is however possible to deduce a constant from the optical rotations for different colours, which is in close relation to the deformation caused by the solvent.

3. This constant is given by the equation of DRUDE [1]), which for an optically active substance indicates the connection between optical rotation and the wavelength of the light for which the rotation obtains. It can be written in the form:

$$\alpha_\lambda = \Sigma \frac{k_a}{\lambda^2 - \lambda_a^2}$$

in which equation α_λ represents the rotation for light of the wavelength λ, while k_a and λ_a represent constants. The constants are controlled by the place of the absorptionbands, and the constants are connected with the forces which resist a displacement of the position of equilibrium of the vibrating system in the molecule which undergoes the influence of the passing light.

For DRUDE, who applied this equation to a few instances, a summation of two terms was sufficient. Afterwards the equation has pretty well fallen into disuse, until LOWRY [2]), in co-operation with many co-workers, pointed out the importance of this equation in a long series of papers. In a great number of cases a single term is sufficient, even for rotations for light from the extreme ultra violet to the infra red, but moreover he made an important application of this equation for the classification of cases of anomalous rotary dispersion, in which at least two terms of DRUDE's equation are necessary in order to be able to represent the measurements.

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[1]) Lehrbuch der Optik, Leipzig 1900.

[2]) J. Chem. Soc. **103**, 1062, 1067, 1322 (1913); **105**, 81, 94 (1914); **107**, 1173, 1187, 1195 (1915); **115**, 300 (1919); **121**, 532 (1922); **125**, 1465. 1593. 2511 (1924); **127**, 238, 604 (1925); C. R. **181**, 909 (1925).

If the rotations satisfy a relation, consisting of one term of DRUDE's equation, when therefore

$$\alpha_\lambda = \frac{k}{\lambda^2 - \lambda_0^2},$$

there is a linear relation between $\frac{1}{\alpha_\lambda}$ and λ^2, so that, either graphically or by calculation, one can, in a certain case quickly ascertain, whether one term will suffice. If this proves indeed to be the case, the values of the constants k and λ_0 can be determined with rather great accuracy. If, however, there is not such a linear relation, it is pretty well impossible to deduce, with any accuracy, values for the four constants k_a, k_b, λ_a and λ_b, which are then necessary, seeing that it is always possible to deduce numerous groups of values of the constants, which offer as close a connection to the data obtained by experiment.

When it appeared that the rotations for pure bornylacetate as function of the light used, could very well be represented by an equation as follows :

$$\alpha_\lambda = \frac{k}{\lambda^2 - \lambda_0^2},$$

we have made it our first object to execute measurements of the rotatory dispersion with the bornylacetate, while this substance was dissolved in different media and in mixtures of them. From these measurements, which could all be represented by equations of one term, we then deduced the values of k, and we have attached our views to the changes which this constant undergoes when the solvent is modified.

4. Though endeavours have not been wanting to calculate the magnitude of the rotation by a theoretical method, at least in a simple case, the result is not very satisfactory. So it is to be understood that the change which a solvent brings about in the rotation cannot be predicted either. When, however, the value of the rotation for a number of colours can be well represented by an equation of DRUDE with only one term, it is possible to say something about the influence of mixed solvents on the magnitude of k. Seeing that this constant is connected with the firmness with which the vibrating system is bound to its position of equilibrium, deformations of the molecule must be reflected in the value of the constant. It is very well possible that the normally appearing deformations have such a slight effect on the value of k, that the accuracy of the polarimetric measurements is insufficient to show those changes with certainty, but we can in the first place expect greater changes in k in those cases in which the choice of the solvent makes a maximal deformation possible. In agreement with our previous views, we shall have to choose a solvent consisting of two components with divergent properties, so that the atmosphere of a molecule of bornylacetate differs in its composition according to the side of the

molecule which we examine; then deformation appears very sharply in consequence of the forces which are called into being by the inclination to restore equilibrium in the composition of the solvent by diffusion.

Each of the four groups which are ranged round the asymmetrical C-atom is therefore under the influence of a differently composed atmosphere, so that the electric moment of these groups is also changed in different ways, with consequently a modification in the elasticity of the vibrating system. When complications do not occur, as chemical binding of the solvent, we may, moreover, expect that the direction of change of k is the same, whatever the antagonistic components of the solvent may be, because the cause of the deformation, mentioned in the preceding paragraph, always acts in the same way.

5. In the preliminary investigations, in which, however, we discussed only the influence on the rotation, not on the dispersion constant, the solvent consisted of water and alcohol. In this case the composition of the medium can be changed only to a certain extent, because, when the amount of water is too great, the ester molecules flee from the medium altogether, and combine to drops. For our actual investigation we have made a somewhat different choice. We decidedly favour the opportunity for deformation when the molecules of bornylacetate must act as link between the component parts of the medium. If they are not completely miscible, while addition of bornylacetate brings about complete miscibility, it is obvious that we can picture the molecule of bornylacetate as the link between atmospheres of strongly divergent composition. A maximal deformation would m.m. occur, when the quantity of bornylacetate was just large enough to bring about complete miscibility. In connection with the small quantity of ester which, in the systems investigated up to this time, was sufficient to bring about the perfect miscibility, which is of course contingent with a slighter rotation of the solutions, we have provisionally let this point rest, and have, in the first place, occupied ourselves with measurements in different solvents and in mixtures of them.

6. Our provisional measurements had shown us that we should have to carry accuracy to the highest standard possible; we have succeeded in the way described hereafter, to determine rotations, also for red and for violet light, which are accurate to within two or three hundredths of a degree.

We obtained light of different colours with the help of the monochromator of VAN CITTERT; in order to get sufficient intensity, especially for red and for violet light, we took as source of light a self regulating arc lamp (of LEITZ, current 4 Amp.). With very closely adjusted slits sufficient light is retained; the narrow spectrum region which is isolated in this way, combined with a half shadow angle of 2°, made a very sensitive adjustment possible.

As instrument we used the polarisation apparatus according to LANDOLT-LIPPICH with triple field of vision. By means of a vernier hundredths of a degree could be read. The polarimeter tube, provided with a double jacket, had a length between the cover glasses of 22.00^5 cm. at 30°.0 C. With the help of a hot-air motor, provided with a little pump, it was possible to pass through the jacket water of constant temperature, coming from a thermostate, which was regulated at 30°.4 C. In the polarimetertube was then a temperature of 30°.0 C. The measurements have always been made for seven colours, pretty regularly distributed over the visible spectrum. In order to facilitate in some cases the comparison with other investigators, we have, as accurately as possible, chosen wavelengths agreeing with strong spectrum lines, so that, when a monochromator is wanting, and sources of light are used which give a line spectrum, combined with prisms "à vision directe", in order to remove the lines that are not wanted, our colours can be reproduced. The correct value of the wavelength of the light used was always checked by measuring the rotation of a standardized plate of quartz; a graduated drum, on which the displacement of the movable slit of the monochromator could be read, made the first rough adjustment of the sort of light, which was wanted, possible; then we corrected this adjustment, until the rotation found, which held good for the plate of quartz, agreed with the calculated value.

We calculated these rotations with the help of an equation for the dispersion of quartz, deduced by LOWRY [1]) out of a huge number of very accurate measurements which stretch far beyond the region of visibility. If the wavelength λ is expressed in microns, the rotation of a plate of quartz, per mm of thickness, is given by:

$$\alpha = \frac{9.5639}{\lambda^2 - 0.0127493} - \frac{2.3113}{\lambda^2 - 0.000974} - 0.1905.$$

Almost perfectly equal results are obtained by the equation:

$$\alpha = \frac{11.6064}{\lambda^2 - 0.010627} + \frac{13.42}{\lambda^2 - 78.22} - \frac{4.3685}{\lambda^2}$$

which LOWRY [2]) had previously deduced from his measurements. Table I gives the wavelengths of the kinds of light chosen, and the corresponding rotations of the normal plate of quartz used.

TABLE I.

Calculated rotation of quartzplate *A* for some kinds of light.

7065	6362	5893	5351	4922	4607	4358 Å.
23°.69	29°.51^5	34°.69^5	42°.59^5	50°.95	58°.78	66°.36

[1]) Philos. Trans. (A) **226**, 391 (1927).
[2]) Philos. Trans. (A) **212**, 261 (1913).

7. The *d*-bornylacetate used came from POLAK's Frutal Works at Amersfoort, and was already pretty pure. When a quantity of about 3 kg had been melted and then crystallized at room temperature a couple of times, until only a small part had remained liquid, we further purified the preparate by letting it, after melting, crystallize at constant temperature in a thermostate. After repeating this process a sufficient number of times, always at slightly higher temperatures, it was possible in this way to obtain at last rather more than 2 L. of bornylacetate, which at 27°.7 still solidified completely.

For the specific volume at 30°.00 C. we found in two determinations: 1.02338, and 1.02340 (d_4^{30} = 0.97715, and 0.97714); at 26°.50 C. the specific volume amounted to: 1.02129 (d_4^{30} 0.97915 respectively 0.97915).

Besides of the pure preparate we have also determined the rotation of two less pure fractions for yellow light (5893 Å). Whereas the rotation in a tube of 22 cm in the first case amounted to 93°.89, we found for a fraction which was crystallized at 0°.1 C. lower temperature 93°.88, and for one which had solidified at 0°.5 C. lower temperature 93°.92, values which are equal within the errors of the experiment.

8. We have now used the purest fraction in order to determine the rotation dispersion of this substance. For this purpose, after regulating the monochromator for the correct colour, we always made 14 readings for equal faintness of the three parts of the field of vision, for the rotation of the plate of quartz as well as for the preparate. From time to time the zero [1]) was also checked; this was found to be constant during the whole investigation. The readings were a little less certain only with the most extreme colours (7065 respectively 4358 Å); yet even then the deviations from the average were rarely greater than 0°.03. For the other colours the possibility of reproduction was much greater, unless fatigue had set in, when long series of measurements had been made.

The results of two fillings (I and II) with the pure preparate, investigated at 30°.2 C., are given in Table II.

In order to get an idea of the influence of temperature on the rotation, and to be able to reduce the figures found to those which hold good for 30°.0 C. we have also investigated preparate II at 26°.6 C. for light of wavelengths: 7065, 5893, 4922, and 4358 Å, at which for the rotations was found 63°.92, 94°.89, 142°.27, and 189°.23. The influence of temperature is therefore relatively small, viz. in the mean 1.06 % for 3°.6 C. (without indication for gradual change of this influence with the colour of the light), or 0.3 % per degree.

In the 5th column Table II contains the values of the rotation, reduced

[1]) We have satisfied ourselves of the fact that the zero did not change by inserting the empty polarimetertube with coverglasses tightly screwed down.

TABLE II.

Rotation of *d*-bornylacetate at 30°.2 C; length of the tube 22.00^5 cm.

λ	Praeparate I	Praeparate II	Mean	α reduced to 30°.0 C.	$[\alpha]_{30.0}$ found	$[\alpha]_{30.0}$ calculated	cal.—found in °/₀₀
4358	187°.15	187°.19	187°.17	187°.28	87°.12	87°.24	+ 1.4
4607	164.14	164.12	164.13	164.23	76.40	76.43	+ 0.4
4922	140.78	140.78	140.78	140.86	65.52^5	65.51	±
5350	116.38	116.37	116.37^5	116.44	54.16	54.15	±
5893	93.89	93.87	93.88	93.94	43.70	43.66	— 0.9
6362	79.34	79.32	79.33	79.38	36.93	36.92	±
7065	63.31	63.22	63.26^5	63.30	29.45	29.46	±

to 30°.0 C., while in column 6 the specific rotations deduced from them are inserted ($d_4^{30°00}$ 0.97714, length of tube = 2.200^5 dm).

9. Supposing that the specific rotation $[\alpha]$ as function of the wavelength λ, can be represented by one term of DRUDE's equation, we can put down seven equations, in order to calculate the two constants appearing in it, of the form [1]):

$$\lambda_0^2 + k\frac{1}{[\alpha]} = \lambda^2$$

these equations contain as unknowns λ_0^2 and k. By solving this set of equations with the help of the method of least squares we find for k and λ_0^2 the values 13.75 and 0.03236 respectively, so that here DRUDE's equation takes the form of:

$$[\alpha] = \frac{13.75}{\lambda^2 - 0.03236}.$$

From the value of λ_0^2 (expressed in microns) follows for λ_0: 1800 Å as the place of the absorption band in the ultra violet.

Column 7 of Table II gives the values of the rotation calculated with this equation and column 8 shows how close the agreement is between the values

[1]) DRUDE's equation slightly transformed.

calculated and the values found. The greatest deviation occurs in the violet, where, on account of the neighbourhood of the absorptionband, the chance of greater differences [1]) can play a part. For the rest we must not lose sight of the fact that the error in the reading of the rotation of the plate of quartz (which shows a three times smaller rotation than the bornylacetate, see Table I) passes 3 × enlarged into the rotation of the preparate. It is seen therefore that the rotation of the bornylacetate can be perfectly represented by one term of DRUDE's equation, a result which is quite in agreement with the results for many other organic compounds investigated by LOWRY.

10. When in this way the constants were therefore determined of pure bornylacetate, we have investigated the preparation in different solvents and have always introduced into the polarimeter-tube solutions which contain 20 grammes of ester (weighed in vacuo) per 100 cc of solution. The rotations, directly measured, then lie between 12° and 42° for red and for violet light respectively. In all the cases investigated it was possible to represent the rotation by one term of DRUDE's equation. The deviations between values found and values calculated rarely surpassed 2 $^0/_{00}$.

Solution in absolute alcohol. The absolute alcohol was prepared by distilling alcohol with quicklime, and contained less than 0.1 % of water

For a solution of 20 % the specific rotations observed can be represented by the equation :

$$[\alpha] = \frac{14.17}{\lambda^2 - 0.03383} \qquad (\lambda_0 = 1839 \text{ Å}).$$

The change in k is therefore very marked, also the change, in $\lambda_0{}^2$, but we shall not take the latter into further consideration, also because slight variations in the values of $[\alpha]$ can cause relatively great changes in $\lambda_0{}^2$. Graphically speaking $\lambda_0{}^2$ indicates the place where the straight line $\left(\frac{1}{[\alpha]}, \lambda^2\right)$, after considerable extrapolation, intersects the axis. A slight change of the coefficient of direction of the line causes a great shifting of this point of intersection.

It may now be expected that the change in k will become greater when more diluted solutions are investigated, because then the molecules of bornylacetate, each for itself, have a greater number of molecules of alcohol at their disposal, or in other words, that their atmospheres do not overlap so much. This proves indeed to be the case ; for a solution of 5 % k rises to 14.86, and for a solution of 1 % to 15.32. These figures however become much less certain on account of the much smaller rotations, so that for the other solvents we have provisionally restricted ourselves to solutions of 20 %.

Now when a solution is prepared which, next to alcohol, contains so

[1]) From the equation follows $[\alpha] = \infty$ for light which is maximally absorbed, as damping is not allowed for.

much water, that at 30°.0 C. this mixture can contain 20 grammes of ester per 100 cc of solution [1]), we have here again a possibility for greater deformation and partial splitting up of the components of the solution at the place of different groups of the estermolecule. Quite in agreement with this is the value of k, which has mounted to 14.58, while for alcohol k is 14.17, and for the pure ester k is 13.75. The solution contained 38.5 % of water (and 61.5 % of alcohol).

11. For a number of solvents and combinations of them Table 3 contains the values of k, which have always been deduced from seven specific rotations for the colours mentioned above. As regards the solvents used we wish to observe :

Pure acetic acid was obtained from glacial acetic acid to which a smaller quantity of acetic anhydride was added than corresponded with the water content, calculated from the initial freezing point. After boiling this for a couple of hours, using a reflux condensor, we again determined the freezing point, which proved to have mounted considerably, and then prepared by fractional freezing an almost pure acetic acid. (Freezing-point 16°.6 C.).

Out of common gasolene two fractions were distilled, one passing between 91° and 108° C. (called "heptane" in the Table), the other of 140°—150° C. (nonane).

Benzene, free from thiophene, was distilled twice from P_2O_5 ; the specific volume at 30°.00 C. was 1.15284 ; boiling point 80°.1—80°.2 C.

We used a commercial preparate decalene, of which a great quantity was kept in stock, which was no further purified ; neither was this done with benzyl alcohol.

After investigating the rotation in pure solvents, we have also dissolved bornylacetate in mixtures of them, and always in the proportion of 1 : 1. When in these mixed media the rotation is a linear function of the composition, the composition must be expressed in mol. percents. The values of k calculated for mixtures which contain equal weights of the two components, are also based on the composition in mol. percents.

Fig. 1 gives a diagram of the values found for k ; Table III gives the figures for k, λ_0^2 and λ_0 in the different solvents.

12. About the dispersion and rotation in the simple media (characterized by k) we may make the following observations. For the solution in the most normal medium we find for k : 13.79, equal to the value which holds good for the pure ester, which is quite in agreement with the conception that in such a case deforming forces do not prevail [2]). The same thing holds good for the hydrogenated, aromatic hydrocarbon decalene.

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[1]) The volume of 100 cc. holds good for 15° C.

[2]) At the same time it must be presumed that the mutual, deforming forces, present in pure ester, are very slight.

TABLE III.

Values of k and λ_0^2 in different media.

Solvent	k	λ_0^2	λ_0 (Å)
Bornylacetate	13.75	0.03236	1800
20 % in aethylalcohol	14.17	0.03383	1839
5 % „ „	14.86	0.02195	1480
1 % „ „	15.32	0.02163	1470
20 % „ alcohol-water	14.58	0.02990	1730
20 % „ acetic acid	14.84	0.03052	1747
„ „ heptane	13.79	0.03277	1810
„ „ acetic acid-heptane 1 : 1	14.94	0.03036	1742
„ „ nonane	13.82	0.03253	1804
„ „ acetic acid-nonane 1 : 1	14.79	0.03108	1763
„ „ benzene	12.72	0.03349	1830
„ „ benzene-acetic acid 1 : 1	14.24	0.02904	1704
„ „ decalene	13.80	0.03067	1751
„ „ decalene-acetic acid 1 : 1	14.74	0.03341	1828
„ „ decalene-alcohol 1 : 1	14.55	0.03154	1776
„ „ benzylalcohol	13.88	0.03388	1840
„ „ benzylalcohol-heptane 1 : 1	14.26	0.03162	1778
„ „ decalene-benzene 1 : 1	13.07	0.03190	1786
„ „ benzene-alcohol 1 : 1	13.84	0.03138	1772
„ „ benzylalcohol-alcohol 1 : 1	14.36	0.02735	1654
„ „ benzylalcohol-decalene 1 : 1	14.29	0.03163	1779
„ „ decalene-heptane 1 : 1	13.79	0.03145	1771
„ „ benzene-benzylalcohol 1 : 1	13.59	0.03419	1849
„ „ benzene-heptane 1 : 1	13.27	0.02855	1690

Benzene, on the contrary, lowers the value of k to 12.72, on the other hand acetic acid gives a rising to 14.84. Evidently benzene (phenylgroup) and acetic-acid act, more particularly, on different parts of the bornyl-acetate (consisting of an aromatic part and an acetate group. Aethylalcohol has an increasing influence on the value of k, while finally a slightly modified value (13.88) appears with benzylalcohol, as a consequence of the presence of a lowering phenyl- and an increasing alcohol-group.

With the mixed media we find results which are quite in agreement with our expectations. The mixing of normal media gives values of k which

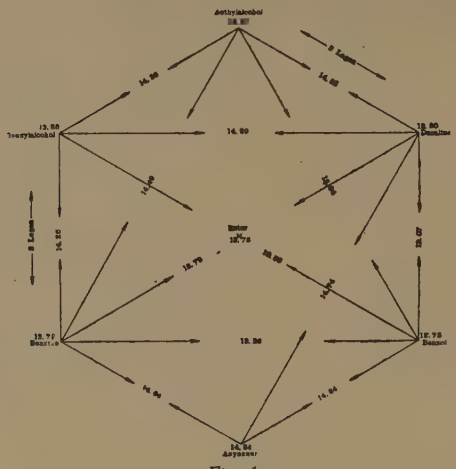

Fig. 1.

deviate but little from those which are calculated according to the rule of three. However, as soon as the components differ more strongly in character,

TABLE IV.

Values of k in mixed media calculated and found.

System	k found	k calculated	Deviation in %	Remarks
Decalene-heptane	13.79	13.79	—	
Decalene-benzene	13.07	13.12	$^1/_2$	
Benzene-heptane	13.26	13.18	$^1/_2$	
Acetic acid-decalene	14.74	14.53	$1^1/_2$	
Acetic acid-benzene	14.24	13.93	2	
Acetic acid-heptane	14.94	14.44	$3^1/_2$	maximum
Alcohol-heptane	14.49	14.05	> 3	„
Alcohol-decalene	14 55	14.08	$3^1/_2$	„
Alcohol-benzene	13.84	13.63	$1^1/_2$	
Alcohol-benzylalcohol	14.36	14.08	2	maximum
Benzylalcohol-heptane	14.26	13.83	> 3	„
Benzylalcohol-decalene	14.29	13.85	> 3	„
Benzylalcohol-benzene	13.59	13.21	< 3	

as f.i. the combinations with acetic acid, much greater deviations of the values calculated appear ; the combination acetic acid-heptane gives a value of k even greater than the value of either of the components and with a deviation of $3\frac{1}{2}$ % from the calculated value ; in the cases investigated it is absent only in the combinations alcohol-benzene, and benzylalcohol-benzene.

Two cases deserve special attention, viz. those in which, without the presence of bornylacetate, a complete mixing of the components does not appear. These are the systems alcohol-decalene, and benzylalcohol-heptane. Here too, the deviation of the calculated value is greater than 3 %, just as has been found in the system acetic acid-heptane, where, at room-temperature, we are in the immediate neighbourhood of the critical solution temperature.

Table IV gives a synopsis of the possible combinations of the six solvents, and the differences between values of k calculated and found. Finally we wish to observe that in all the cases where the deviations are not too small, the values found are greater than the values calculated.

Summary.

After having demonstrated that the rotation dispersion of bornylacetate as a liquid and in the dissolved state can always be represented by one term of DRUDE's equation, the value for the constant k, which appears in it, has been determined for six solvents, and the possible binary combinations of them. The variations in the value of k are quite in agreement with the views we have developed in a previous paper on the role of mixed media with respect to a dissolved substance.

Utrecht, October 1928. VAN 'T HOFF-*Laboratory.*

Anatomy. — *The Development of the dental system in Trichosurus vulpecula.* By L. BOLK.

(Communicated at the meeting of December 22, 1928).

By courtesy of Dr. HILL from London I obtained a series of marsupial young from Trichosurus vulpecula in various periods of development. This material is very suitable to the study of the development of teeth, since the various stages followed each other closely. This circumstance may be necessary for every research into the development of teeth, it is specially so with marsupials, since phenomena of high importance do change here very rapidly, therefore are only observable during a limited phase of the development. In the literature on the dental system of marsupials statements of great importance are sometimes found, based on insufficient material.

An extensive report on the results of this research will be published elsewhere, in this communication only a few remarkable phenomena in the development of teeth in Trichosurus vulpecula will be set forth, and a brief discussion of the justifiable results will be given.

A summary of the odontological points of view from which I look on the development of teeth in marsupials and of the questions I tried to solve in this research will now be given in order to insure a clear understanding. For in regard to those points I differ from current opinions. As a starting point I mention the fact that the marsupials differ from monodelphous mammals by the absence of a tooth-change: they are monophyodont as contrasted with the diophyodontism of the latter group. This monophyodontism is not absolute: as a rule a single tooth is changed. This peculiarity in the teeth of marsupials did arise the question which of the two sets of teeth of the monodelphians is homologous with the marsupialian set: the deciduous or the permanent teeth.

Concerning this question the opinion of the authors varies widely, a group regarding the marsupialian teeth as homologous with the deciduous set of the monodelphians, another group to the permanent set of teeth. After both opinions one of the sets should have been lost by a process of reduction: the monophyodontism should have arisen out of diphyodontism by the loss of one set of teeth. (It must be stated that LECHE regards the diphyodontism of monodelphians as a property acquired by this group.)

It is evident that the opinion regarding monophyodontism as a phenomenon of reduction is the logical application of the hypothesis explaining the diphyodontism in mammals by a gradual decline in the numerous dentitions in lower vertebrates. Therefore, di- and polyphyo-

dontism should be phenomena differing only gradually, and monophyodontism should be the result of a progression of this process.

In former publications [1]) I have opposed myself against this general opinion, since I am convinced by my researches that the changing of teeth in mammals differs qualitatively from that in lower vertebrates — specially in the reptiles — as will become apparent from the following summary.

In mammals the dental lamina gives rise to two rows of dental germs, an outer row or exostichos, an inner row or endostichos, the former developing into the deciduous set of teeth, the latter into the permanent. When the teeth change, the exostichal teeth, which are formed earlier and develop quicker, are pushed out by the endostichal. The tooth changing in monodelphians therefore bears the character of a consecutive appearance of two rows of teeth, and it is clear that in those animals only once a change can take place. The dental system in this group might be characterized by the term "substitutive-dentition".

In reptiles the process of changing has a quite different character. Here also the teeth originate from two rows, but in contrary to the monodelphians the elements of both rows develop simultaneously. During further development each tooth from the inner row moves to an interdental space between two teeth from the exostichos and participates in the composition of the actual functioning set. As contrasted with the dental system of mammalians, that of reptiles can be designated as "intercalated dentition". Whereas the functioning set of teeth in monodelphians is composed either of endostichai or of exostichal teeth only, the dental system of reptiles consists of alternating exo- and endostichal elements.

It is irrelevant that this regularity can be disturbed in the course of life. The phenomenon of intercalation is in itself sufficient proof that the process of changing in reptiles is absolutely different from that in monodelphians. In order to understand the changing process in lower vertebrata it must be born in mind that the teeth in the dental lamina originate from a matrix. In mammals the generating power of the matrix is exhausted after the production of a single tooth ; on the other hand, in lower vertebrata the matrix keeps on forming teeth in intervals. Each matrix in those animals produces, what I have called a family of teeth. The changing process in polyphyodontal vertebrata consists in the periodical elimination of the functioning member of the family by the following younger member. Therefore the changing of teeth can go on unlimited during the life of those animals, in the same way as scales and hairs.

Summarizing : the process of the changing of teeth in mammals is an elimination of the exostichal row and a substitution of this row by the endostichal teeth, in reptiles it consists in an expulsion of the oldest member of a family by the next one. Diphyodontism therefore is not a reduced polyphyodontism ; both processes are fundamentally different.

[1]) Odontological Essays. V. On the Relation between reptilian and mammalian dentition. Journ. of Anat. Vol. 57. 1922.

If this be understood the problem of monophyodontism of marsupials obtains a different aspect since several new possibilities arise. One can suppose that the recent marsupials originate from diphyodontal mammalian ancestors, with a milk- and a permanent dentition, one of these rows, exostichos (milkdentition) or endostichos (permanent dentition), being reduced. Reasoning from this hypothesis the question with which of these rows the dental system of recent marsupials is homologous, is justified. But this hypothesis is not very probable. One is inclined to expect that such a primitive group of mammals as the marsupials are, should show the first and still incomplete indication of the new mechanism of teeth changing acquired by mammals, instead of phenomena indicating a complete mechanism markedly reduced. The well-known change of a single tooth in marsupials can be regarded as an indication of the beginning of such a diphyodontism.

The primitive nature of marsupials gives rise to another question, i.e. whether the changing of teeth does not occur because the structure of the dental system is still like that in reptiles, composed of teeth of the endostichal and exostichal row, or using the monodelphian terminology : of deciduous and permanent teeth. If this be true, this primitive group of mammals should possess matrices forming a single tooth, just as in higher mammals, and on the other side they should have the phenomenon of intercalation in common with the reptiles.

In this way the marsupials should represent also in their dental system a link between the reptilian-like ancestors of the mammals and the highly developed class of monodelphians.

A formerly executed research into the dental system of Perameles had made it clear to me that the dental germs are ranged on the dental lamina in two rows, an endostichal and exostichal one, identical with the deciduous and permanent set of monodelphians ; that in both rows several germs reduce ; and that the functioning dentition is formed by the completely developed germs of both rows. The dental system of this marsupial is an intercalation system. By this structure a changing of teeth in this animal is wholly excluded, the matrices forming a single tooth only. In Fig 1 a diagram

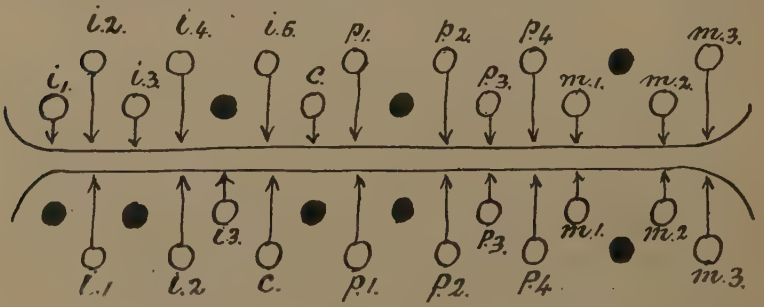

Fig. 1.

is given showing the origin of teeth in this animal in relation to both rows. The black dots indicate germs that do not develop. This figure needs no further explanation ; particulars are found in my paper on this question [1]).

In my research on the ontogenesis of the dental system of Trichosurus special attention was given to this point, but I have not found any indication of intercalation in this very markedly reduced dental system. The explanation is very simple ; the dental lamina is build so irregularly, whereas the dental germs appear in such an irregular manner and such different intervals, that it is impossible to recognize an arrangement in two rows. However, during the development conditions may arise that seem to prove a distichal distribution of the tooth germs and an intercalation of the elements of these two rows, as may be proved by Figure 2. This figure represents a reconstruction of the dental germs in the upper jaw of a pouch young of Trichosurus, measuring in total 58 mm. There exists no regular succession in posterior direction in the enamel organs as is the case with embryos of monodelphians. Four of the germs might be regarded as forming an outer row, and three as forming an inner row. I do not want to exclude the possibility that the arrangement of the dental organs in figure 2 is really caused by a distichal origin of the dental system, and that therefore in Trichosurus also this system is composed of "deciduous" and "permanent" teeth, but I cannot consider this case as conclusive proof, since the arrangement of the dental germs might be caused in other ways. It is evident that the marked reduction of the number of teeth of Trichosurus renders this specimen unsuitable for an inquiry into the phenomenon of intercalation.

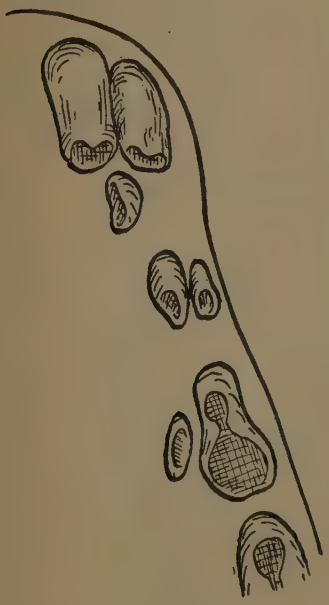

Fig. 2.

In regard to another question I obtained a more positive result. The ontogenesis of the dental system of Trichosurus does not procure any indication in favour of the hypothesis that ancestors of this animal should have been diphyodontal in the sense of the monodelphians. If one assumes the occurrence of a regular diphyodontismus in the ancestors of marsupials, it is possible to interprate certain abortive germs as belonging to the first, others to the second dentition. But this point of view is purely aprioristic. An objective research into the correctness of this hypothesis fails to yield

[1]) Die Beziehungen zwischen Reptilien- Beutler- und Plazentaliergebisz. Zeitschr. f. Morphologie und Anthropologie. Bnd. 20. 1917.

proof in favour of it. The changing of the so-called fourth praemolar of the marsupials is the indication of a beginning diphyodontism.

In regard to structure and morphological significance of the dental system of Trichosurus, my research of its development did not yield the results I expected, the intercalation of both rows, so evident in Perameles, not being observable in Trichosurus, by cause of the considerable reduction of the number of teeth.

On the other hand, in regard to another fundamental odontological problem, viz. the morphological structure of the tooth, this development yields very important data. In order to make their significance clear I must summarise shortly the quintessence of my theory on the dimery of the mammalian tooth. After this theory each mammalian tooth corresponds with two consecutive members of a tooth-family of the reptiles.

The matrices in the dental lamina of these vertebrates, produce — as pointed out already — teeth during the entire life, two products are separated by a longer or shorter period of rest, each tooth therefore is anatomically independent. In mammals this process has been changed in two senses. In the first place the function of the matrix is limited to the production of a single tooth, but also — and this is the second change — this process includes the formation of two tooth-germs. The activity of the matrix therefore is not only very much decreased, also concentration has taken place ; the period of rest between two activities is eliminated, and the product therefore is a double-tooth ; the relief of the crown still showing the limit between the two components. For the buccal row of tubercles represents the older generation — the protomer —, the lingual the younger — the deuteromer. The polyphyodontism of reptiles finds therefore still its expression in the relief of the crown of mammalian teeth. Such a tooth represents two generations of a tooth-family of reptiles.

Now, in the ontogenesis of mammalian teeth two phenomena occur that are connected with the origin of teeth as set forth here, i.e. the double connection of the enamel organ with the dental lamina and the originating of the pulpa on two centers of the organ : one in the buccal and one in the lingual half. Temporarely those centra of pulpa formation are separated by a layer of indifferent cells : the enamel septum. Those phenomena can be observed very clearly in the course of development of teeth of Trichosurus, but I do not enter further into this question since I am unable to communicate anything new. On the other hand I have observed some peculiarities in the ontogenesis of those teeth that I have not seen in any other mammal and that yield new support to the correctness of the dimer theory of the mammalian tooth. The description of the following facts may make this clear.

The dental system of Trichosurus is markedly reduced, and certainly not a few teeth, present in its ancestors, are lost. This is sufficiently proved by the fact that during the development several dental germs originate that do not further develop. Some of those exist for a short period only and are quickly resorbed, others remain for a longer time. Fifteen developmental stages of

the dental system have been investigated by me and a compilation of all dental germs observed in upper and lower jaw is given in Fig. 3.

Those that develop regularly have been striped ; all other germs disappear sooner or later even if dentine substance has been formed already. The

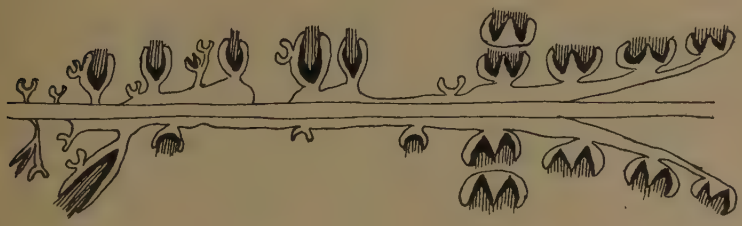

Fig. 3.

abortive germs ask our special attention. The diagram in which their topographical relations are given as accurately as possible, shows that they may arise : *a*, directly from the epithelium of the jaw ; *b*, from the dental lamina ; and *c*, from a dental germ that develops itself into a complete tooth. Those from the last group are the most important from an ontogenetic point of view. The occurrence of such a rudimental germ arising from another has been observed by me in the upper jaw in the germs of the medial incisors and canines, in the lower jaw in the germ of the incisor. They appear without exception in an early stage of development before the germ from which they arise has been invaginated. During my odontological researches this is the first time I found such conditions and their great rarity certainly is a sign of their uncommon significance. It is not difficult to understand their meaning if we consider the facts from the point of view of the dimer theory.

We must start from the fact that we meet here single tooth-germs showing tendency to develop two teeth, one of which develops regularly, the other being resorbed. It is clear that they do not represent a deciduous tooth and its substitute, for in mammals those two arise never from a single germ and simultaneously. The only acceptable explanation is that in those cases the dimer origin of the mammalian tooth is demonstrated, each germ corresponding with an odontomer, of which only one develops completely. The historical course of development of the mammalian tooth in those cases takes a more or less reversed direction : two independent tooth-germs which once merge into a single formation now are separated again. If both separated products should develop completely, two teeth should appear, placed side by side as a buccal and lingual one. But one of those germs atrophying very soon, a single tooth develops completely, which however is not a dimer element, not aequivalent to a mammalian tooth, because it is monomer like a reptilian tooth.

With this last conclusion the importance of the phenomenon described

in favour of the theory of dimery is expressed. And the correctness of this conclusion is proved by the following facts.

Whereas the presence of the ontogenetic symptoms of dimery — enamel septum and double connection with dental lamina — could be ascertained in the development of all teeth of Trichosurus, they are absent in the three teeth with abortive adhaerent germ. This fact proves that it was correct to diagnose this germ as one of the odontomers, probably the deuteromer. The medial incisor and canine in the upper jaw as well as the incisor in the lower jaw are therefore monomer teeth. This peculiarity in the ontogenesis of the indicated teeth probably explains the following fact. The medial incisor of the upper jaw has a crown with a perfectly smooth lingual surface, whereas this surface in the second and third incisor shows a tubercle, oblong in shape and bordered by a deep groove that reaches the edge. Therefore in the latter teeth the dimery is witnessed by the relief of the crown, bearing a lingual and buccal tubercle, in the medial incisor on the contrary no such indication can be found.

In regard to the historical development of the mammalian tooth the investigation of Trichosurus has yielded more important data than in regard to the structure of the dental system ; they support the hypothesis of dimery.

During the development of the teeth of Trichosurus still another phenomenon occurs, that yields an argument for this theory in no less convincing manner. As Fig. 3 shows, a number of rudimentary germs appear, arising either directly from the epithelium of the mouth or from the dental lamina.

We will not enter here into a discussion upon all these germs, but the attention may been drawn to two of those germs, i.e. the most medial in the lower jaw and that between third and fourth incisor in the upper jaw. Those show an extremely peculiar development from which Fig. 4 offers a diagram.

Fig. 4.

From the dental lamina, a string arises bearing a rudimental tooth germ, the papilla of which is covered by a thin layer of dentine. Probably this germ represents a tooth reduced relatively not long ago, an incisor functioning yet in a rather recent ancestor of Trichosurus. The peculiarity is that after forming the first rudimental germ the stem that connects the germ with the dental lamina proceeds and ends in a second tooth-germ. This germ also is invaginated but there is no dentine substance. An absolutely aequal development is shown by the most medial rudimental tooth germ in the lower jaw as is shown in Fig. 3. A situation as represented in Fig. 4 was never seen by me in any mammalian, but the conditions of

development in reptilian teeth are immediately called to mind. The diagram might have been taken from a section through the embryonic dental lamina of a reptile, the dental matrix having produced the first member of the tooth-family and now starting to form the consecutive member.

This comparison with the manner in which the dental system of reptiles develops is made on purpose, since I am convinced that in Fig. 4 an aequivalent and not an analogous process is represented. For what is the case? We are concerned with reduced mammalian teeth of which the phylogenetic development is followed in reversed direction; the dimer organ, once formed by the concentration of two dental germs, is again divided into its components. Both germs from Fig. 4 therefore represent the odontomers of a normal tooth and in the first of the two even dentine has been formed. Fig. 4 therefore represents a historical stage in the development of the mammalian tooth; if both germs are conceived to be in anatomical connection we have a representation of the dental germ of a mammalian tooth.

The circumstance that it was possible to demonstrate the anatomical evidence of a double generating power in mammalian teeth in two reduced teeth of Trichosurus vulpecula, is of no little importance, since it excludes the possibility of accidental variation.

Botany. — *Die Funktion der Alkaloide in den Blättern von Cinchona succirubra Pavon.* Von TH. WEEVERS und H. D. VAN OORT. (Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of December 22, 1928).

Die Alkaloide der Cinchonablätter sind schon einige Male das Objekt pflanzenphysiologischer Forschungen gewesen. Im Jahre 1897 hat J. P LOTSY [1]) eine Arbeit veröffentlicht, in welcher er die Hypothese von DE VRY [2]) durch Experimente zu stützen versuchte, während im Jahre 1910 P. VAN LEERSUM [3]) in den „Proceedings" eine hierauf bezügliche Mitteilung angeboten hat. In letzterer kam dieser Autor jedoch zu Folgerungen, die denjenigen LOTSYS schnurgerade zuwider liefen.

Die Hypothese DE VRYS war, dass die Cinchona-alkaloide in den Blättern gebildet und aus diesen Teilen nach der Rinde transportiert würden und damit eine Umbildung der amorphen Blattalkaloide in die kristallinischen Alkaloide Chinin mit seinen Nebenalkaloiden Cinchinin, Cinchonidin und Chinidin stattfinde. LOTSY behauptete, dass die Bildung in den Blättern als direkte und indirekte Folge der Assimilation am Tage vor sich gehe. Weil jedoch die von ihm benutzte Methode zu quantitativen Untersuchungen völlig ungeeignet war, eine Tatsache, die schon VAN LEERSUM hervorgehoben hat, kann den Ergebnissen LOTSYS keine Beweiskraft zuerkannt werden.

VAN LEERSUM benutzte die nach zahlreichen Untersuchungen als tüchtig anerkannte Methode, n.l. Aether-Extrahierung der pulverisierten und mit gebranntem Kalk versetzten Teile. Der eingeengte und mit Wasser versetzte Extrakt wurde sodann von Beimischungen gereinigt, indem er zuerst schwach alkalisch gemacht und dann mit Aether ausgeschüttelt wurde. Die aetherische Lösung der Alkaloide schüttelte er wieder mit angesäuertem Wasser aus und bestimmte in diesem wässerigen Extrakt die Alkaloide durch Titration. (Indikator Haematoxylin). Eine Schwierigkeit, die der Genauigkeit dieser Methode Eintracht tut, ist, dass der Umschlagpunkt höchstens bis 0.1—0.2 cc genau zu bestimmen ist.

Setzen wir das Molekulargewicht der Blattalkaloide auf 300, so stimmt 0.15 cc mit 4.5 mg überein, bei Analysierung von 20 g Trockengewicht liegt die Fehlergrenze also bei 0.02 %. Wichtiger ist jedoch, dass VAN LEERSUM bei der groszen Mehrzahl seiner Versuche die Alkaloide pro hundert des Trockengewichts berechnet, ein Verfahren, das zu fehlerhaften

[1]) J. P. LOTSY. Mededeeling 's Lands Plantentuin 1899.
[2]) J. E. DE VRIJ, Ned. Tijdschrift v. Pharm. Chem. en Toxicolog. 1899.
[3]) P. VAN LEERSUM, Proc. Kon. Akad. v. Wetensch. Amsterdam, 1910.

Folgerungen Veranlassung geben muss. Wenn z. B. das Trockengewicht der Blätter bei Versuchen im Dunkeln von 10 g bis 8 g abnimmt und der Gehalt von 1.8 bis 2.0 % steigt, so hat die absolute Quantität dennoch abgenommen und zwar von 180 bis 160 mg. Also haben blosz diejenigen Versuche Wert, in welchen eine Vergleichung der absoluten Quantitäten durchgeführt worden ist und zwar in mit einander gut vergleichbaren Teilen, wie rechten und linken Hälften einer nicht zu geringen Anzahl symmetrischer Blätter.

Blosz einige der Versuche VAN LEERSUMS entsprechen diesem Erfordernis, das heiszt die, in denen die Alkaloidquantitäten in 50 Blatthälften verglichen wurden. Die Kontrollhälften wurden morgens früh von den Mittelnerven abgeschnitten, die anderen abends. In seiner Mitteilung erwähnt VAN LEERSUM drei derartige Versuche ; bei zwei hatten die Alkaloide am Tage zugenommen, resp. von 197 bis 212 mg. und von 248 bis 254 mg., beim dritten Versuch war jedoch eine Abnahme von 235 bis 217 mg zu konstatieren, im Durchschnitt eine Abnahme von 226 bis 224 mg, also Werte die zweifelsohne innerhalb der Fehlergrenze liegen, sodass u. E. nicht erlaubt ist, aus diesen Versuchen Folgerungen zu ziehen.

Etwas besser steht es um den Versuch, wo Blätter 14 Tage an der Pflanze in Stanniol gehüllt blieben und dann die Hälfte jedes Blattes als Kontrolle abgeschnitten wurde, später die andern Hälften analysiert wurden, nachdem die Pflanze wiederum 14 Tage im Lichte gestanden hatte. Die Alkaloidquantität hat dann von 214 bis 198 mg abgenommen, eine Abnahme aus welcher VAN LEERSUM folgern will, dass die Alkaloide nicht bei der Assimilation entstehen. Im Gegenteil betrachtet er sie ebenfalls im Zusammenhang mit den Experimenten, worin blosz prozentische Werte bestimmt wurden, als Dissimilationsprodukte.

Sowie schon hervorgehoben wurde, können wir letzteren wenig Wert beimessen, während bei den zuerst in Stanniol gehüllten Blättern es sehr die Frage ist, ob diese im feuchten Tropenklima Westjavas nicht durch dieses Verfahren endgültig beschädigt würden. Ueberdies enthält VAN LEERSUMS, Arbeit u. E. einen Widerspruch, indem er sagt : 1^0. die Blattalkaloide entstehen bei den Dissimilationsprozessen in den Blättern. 2^0. diese Stoffe werden nicht transportiert und kehren ebensowenig in den Stoffwechsel zurück. Die logische Folgerung ist, dass die Blattalkaloide sich im alternden Blatte anhäufen müssen ; die Behauptung VAN LEERSUMS, dass abfallende Blätter ebensoviel Alkaloide als erwachsene enthalten, ist jedoch völlig damit im Widerspruch.

Durch die Annahme, bei Alkaloidspaltung entstehe Ammoniak, das aus den Blättern entweiche, wäre es möglich diesen Widerspruch zu lösen. Bis vor kurzem klang diese Annahme wenig wahrscheinlich, nach Veröffentlichung der Arbeit von G. KLEIN [1]) muss jedoch auch dieser Erklärung Rechnung getragen werden.

Obenstehendes veranlasste uns die Funktion der Blattalkaloide im

[1]) G. KLEIN und M. STEINER, Jahrb. f. wiss. Botanik. 1928.

Stoffwechsel Cinchonas aufs Neue zu studieren. Dabei beschränkten wir uns vorläufig auf das Studium der Frage, ob die amorphen Blattalkaloide als Dissimilationsprodukte betrachtet werden sollten, später musste dann geprüft werden ob entweder an Anhäufung dieser Produkte am Bildungsort, oder an Transport zu denken wäre.

Die Arbeit MOTHES [1]) über Nikotin lehrt uns, wie verwickelt die Sachlage sein kann, aber GADAMER [2]) und später SABALITSCHKA [3]) hatten vollkommen Recht, als sie sagten, die Frage nach der Bedeutung der Alkaloide im Allgemeinen sei unzweckmäszig. Die Frage soll lauten: welche Bedeutung hat eine bestimmtes Alkaloid für den Stoffwechsel der betreffenden Pflanze; in diesem Falle ist Verallgemeinerung unrichtig.

Unsre Versuche wurden mit jungen, im Treibhause des Amsterdamer Pflanzengartens gezogenen *Cinchona succirubra* Pflanzen angestellt. Einige Pflanzen waren 3, andere 6 Jahre alt, alle hatten gut ausgewachsene 12—20 cm grosze Blätter und die Versuche fanden vom August bis zum Oktober bei einer Temperatur von 23—30° C. im Treibhause statt. Meistens wurden 20 erwachsene Blätter zu den Versuchen benutzt, die eine Hälfte wurde beim Anfang die Mittelnerven entlang abgeschnitten, während die andere Hälfte am Ende des Versuchs in derselben Weise behandelt wurde. Vorversuche zeigten, dass 20 rechte und linke Cinchona-blatthälften in bezug auf ihr Trockengewicht und auf ihre Alkaloid-quantität als gut vergleichbare Gröszen zu betrachten sind.

Die von uns benutzte Bestimmungsmethode ist die G. VAN DER SLEENS [4]).

Die Methode ist folgende:

Die Blätter werden bei 105° C. schnell getrocknet und pulverisiert. 2 bis 3 g. dieses Pulvers mischt man mit (0.5 g.) ungelöschtem Kalk und einigen Tropfen Ammoniak. Das Pulver wird in einem vereinfachten Soxhlettapparate 8 Stunden mit 25 cc. Aether extrahiert. Der Extrakt wird durch ein Wattebäuschen filtriert und einmal mit 1 cc. Eisessig dann 3 × mit 2 cc. Aether nachgespült. Nun wird die ätherische Lösung zuerst mit 10 cc. Wasser, dann 3 × mit 10 cc. angesäuertem Wasser (3 cc. 25 % H Cl pro L.) ausgeschüttelt, sodass alles Alkaloid in das Wasser übergeht. Dem wässerigen Extrakt wird ± 50 mg. Norit hinzugefügt und durch Erwärmen auf dem Wasserbade der Aether verdunstet. Die wässerige Lösung bringt man nach Filtrieren in einen Scheidetrichter und fügt nach Abkühlung 10 cc. Aether hinzu, schüttelt kräftig und macht durch Ammoniak die Reaktion schwach alkalisch. Abermals wird 4 × mit 10 cc. Aether ausgeschüttelt, sodass alles Alkaloid in den Aether übergeht,

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[1]) K. MOTHES, Pflanzenphysiol. Untersuchungen über die Alkaloide. Planta 1928.

[2]) J. GADAMER: Ueber die biologische Bedeutung und Entstehung der Alkaloide. Ber. d. deutsch pharmazeut. Gesellschaft. 1914

[3]) TH. SABALITSCHKA, Die Pflanzenphysiol. Bedeutung der Alkaloide. Ber. d. deutsch. pharmazeut. Gesellschaft. 1923.

[4]) Dr. G. V. D. SLEEN benutzte die Methode bei Arbeiten über die Blattalkaloide im Kina Bureau, Amsterdam.

wie mit Jodjodkalium geprüft werden kann. Dann destilliert man den Aether ab und bestimmt den Rest nach 36 stündigem Stehen im Vakuumexsikkator durch Wägung. Ein Nachteil dieser Methode ist, dass Spuren eines gelben Farbstoffs mitgehen und als Alkaloid gewogen werden; übrigens stimmen die Ergebnisse der stets im Duplo vorgenommenen Versuche gut überein.

Zuerst verglichen wir die Alkaloidmengen pro Gramm Trockengewicht der abgefallenen und der erwachsenen Blätter. In ersteren war die Alkaloidmenge pro 25 Blätter 17.3 mg., in letzteren (Blätter derselben Pflanzen) 17.8 mg. Weil jedoch das Trockengewicht der abgefallenen Blätter pro cm^2 4.2 mg, das der erwachsenen 5.1 pro cm^2 ist, hat das Total der Alkaloide pro cm^2 beim Altern abgenommen, obschon diese Abnahme nicht gross zu nennen ist, ca 10 %.

Dann folgten einige Versuche, bei welchen die Kontrollhälften morgens 8 Uhr, vom Mittelnerven abgeschnitten wurden; die anderen blieben bis abends 8 Uhr an der Pflanze und wurden dann analysiert.

1er Versuch morgens 1.56 % (auf Trockengewicht berechnet)
abends 1.53 %
2er Versuch morgens 1.69 %
abends 1.75 %
3er Versuch morgens 1.35 %
abends 1.45 %
Im Durchschnitt also morgens 1.53 %, abends 1.57 %.

Weil unter diesen Umständen das Trockengewicht pro 10 Blatthälften von 4.4 g. bis 4.6 g. zunimmt, hat die durchschnittliche Totalquantität der Alkaloide von 67 bis 72 mg. zugenommen, eine so kleine Zunahme, dass sie die Grenzen des mittelbaren Fehlers kaum überschreitet. Dass diese Zunahme am Tage stattfindet, deutet durchaus nicht darauf hin, dass die Alkaloidbildung mit den Prozessen der Synthese in Zusammenhang zu bringen sei, wie später sich ergeben wird.

Bei einer zweiten Versuchsreihe wurden die Kontrollhälften abends 8 Uhr abgeschnitten; die anderen blieben bis zum folgenden Morgen 8 Uhr an den verdunkelten Pflanzen und wurden dann abgeschnitten und analysiert.

4er Versuch abends 2.29 %
morgens 2.43 %

Das Trockengewicht von 10 Blatthälften hat unter diesen Umständen von 4.300 bis 3.900 g. abgenommen, die Alkaloidquantität nahm deshalb zu von 94 bis 95 mg. eine Zunahme innerhalb der Fehlergrenze.

5er Versuch abends 2.20 %
morgens 2.06 %

Das Trockengewicht pro 10 Blatthälften hat von 3.050 g. bis 2.930 g. abgenommen, die Alkaloidquantität von 67 bis 60 mg. [1])

Dann stellten wir einige Versuche an mit abgepflückten Blättern, deren eine Hälfte beim Anfang der Versuche den Mittelnerven entlang abgeschnitten wurde, während die anderen Hälften in einem dunklen Raum des Treibhauses mit den Stielen in Wasser gestellt wurden.

6er Versuch abends 6 Uhr 1.91 %
morgens 8 Uhr 2.33 %

Beim Aufbewahren während der Nacht hatte das Trockengewicht von 4.900 bis 4.800 g. abgenommen, die Totalquantität der Alkaloide also von 94 bis 113 mg. zugenommen.

7er Versuch abends 6 Uhr 1.87 %
Nach 120 stündigem Verdunkeln 2.32 %

Trockengewichtsabnahme von 4.800 bis 4.400 g.
Zunahme der Alkaloide von 90 bis 112 mg.

8er Versuch abends 6 Uhr 1.33 %
Nach 180 stündigem Verdunkeln 2.14 %

Trockengewichtsabnahme von 7.250 bis 6.550 g.
Zunahme der Alkaloide von 96 bis 140 mg.

Die letzten Versuche ergaben eine deutliche Alkaloidzunahme. Dabei muss erwähnt werden, dass die Blatthälften am Ende der Versuche gar keine sichtbaren Zeichen des Ablebens ergaben, sie waren grün und turgeszent. Im Dunkeln müssen die Dissimilationsprozesse die Oberhand bekommen, obschon für die Annahme, dass die Dissimilation dann abnormal intensiv sei, keine Gründe vorliegen.

Sobald also die Dissimilation die Oberhand hat und Transport unmöglich ist, denn ins Wasser treten keine Stoffe über, zeigt sich eine deutliche Zunahme der Blattalkaloide. Es liegt deshalb auf der Hand diese Stoffe als Dissimilationsprodukte zu betrachten, eine Hypothese, die mit der Meinung von VAN LEERSUM übereinstimmt. Ob dabei speziell an Eiweissdissimilation zu denken sei, ist eine Frage die vorläufig noch nicht für Beantwortung zugänglich ist [2]). Beim dritten Versuch entspricht eine Alkaloidzunahme von 44 mg., einer Eiweiszabnahme van ± 100 mg. (Eiweiszbestimmung mit der Methode STUTZERS). Der 6e Versuch zeigte, dass schon nach einer Nacht die Alkaloidzunahme sehr deutlich sein kann.

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[1]) Der Alkaloidgehalt der Mittelnerven war 1.92 %, pro 10 Exemplare ist die Totalquantität 20 mg.

[2]) Der Hypothese PICTETS gemäsz, soll die Indolgruppe des Eiweiszes unter Einwirkung von Formaldehyd methyliert und dann zum Chinolinkern umgebildet werden.

falls der Versuch mit abgeschnittenen und inWasser gestellten Blatthälften vorgenommen wird. Weil nun bei den an der Pflanze belassenen Blatthälften die Alkaloidquantität unverändert blieb oder sogar abnahm, ist die Folgerung erlaubt, dass dieser Unterschied, dem Umstand, dass dann durch Verbindung mit dem Stamme Transport möglich ist, zu verdanken sei.

Wie dieser Transport stattfindet und in welcher Weise er hervorgerufen wird, ist noch völlig unbekannt, aber dass die Hypothese von DE VRY die richtige ist, liegt auf der Hand. Bei Annahme dieser Hypothese wird auch deutlich, wie es möglich ist, dass in abfallenden Blättern die Alkaloidquantität kleiner ist als in erwachsenen. Spaltung der Alkaloide mit Ammoniakbildung kann selbstverständlich hier nicht die Ursache sein, sonst würde die Abnahme ebenso gut bei den in Wasser gestellten Blättern stattfinden.

Zusammenfassend können wir also aus obenstehendem schliessen, dass die Blattalkaloide Cinchonas bei der Dissimilation entstehen.

Die Dissimilationsprozesse finden in den Blättern fortwährend statt wie in bezug auf die Eiweiszdissimilation durch die Untersuchungen von RUHLAND [1]) und MOTHES [2]) annehmlich gemacht worden ist und wie in bezug auf die Xanthinderivate enthaltenden Pflanzen von einem von uns schon früher erörtert worden ist [3]).

Hier bei *Cinchona succirubra* bringen obenstehende Versuche uns zu der Folgerung, dass die Blattalkaloide nach dem Stengel transportiert werden und vielleicht eine Umbildung in die kristallinischen Alkaloide Chinin, Chinidin, Cinchonin und Cinchonidin dabei im Spiele sei.

Die Frage ob, wie die Xanthinderivate Koffein und Theobromin auch die Alkaloide Cinchonas wiederum in den Stoffwechsel zurücktreten können, muss noch genauer ins Auge gefasst werden.

In Anbetracht der Arbeiten von VAN LEERSUM, KERBOSCH und SPRUIT ist dies jedoch nicht wahrscheinlich. Ihre Versuche bezogen sich auf die Frage, ob die Alkaloide in der Rinde beim Altern der Bäume in Quantität zunehmen, eine Frage deren Beantwortung sehr schwierig ist. Vergleichung der prozentischen Werte kann schwerlich zum Ziele führen, weil die oft beobachtete Abnahme der Werte pro hundert Trockengewicht blosz eine relative Abnahme belegt, die durch stärkere Zunahme der Bastelemente hervorgerufen werden kann.

Die Ringelungsversuche von VAN LEERSUM belegen nur wenig in bezug auf die Ab- oder Zunahme der Alkaloide. Wenn dieser Autor konstatiert, dass die prozentischen Werte der Alkaloide 6 Wochen nach der Ringelung 7.85 % und 12 Wochen später 6.67 % sind, so besagt das nichts in bezug auf die absoluten Quantitäten. Ueber der Ringelungsstelle häufen

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[1]) RUHLAND und WETZEL, Die Wechselbeziehungen in Stickstoff und Säurestoffwechsel. Planta 1921.

[2]) K. MOTHES, Beitrag zur Kenntnis des N. Stoffwechsels höherer Pflanzen. Planta 1926.

[3]) TH. WEEVERS. Die physiologische Bedeutung des Kaffeins und des Theobromins. Annales du Jardin botanique de Buitenzorg. Vol. 6, 1907.

sich in der Rinde, die aus den Blättern zuströmenden Nahrungsstoffe an. Hier muss deshalb das Trockengewicht zunehmen und wie schon oben hervorgehoben wurde, kann eine Alkaloidabnahme pro hundert sehr gut mit absoluter Zunahme verknüpft sein.

Die Versuche von KERBOSCH und SPRUIT [1]) bei welchen die Totalquantität der Alkaloide pro Bastring (1 m breit und 1 m über dem Boden) berechnet wird, lehren, wie wichtig diese auch für die Praxis sein mögen, uns nur wenig in pflanzenphysiologischer Hinsicht. Ihre Ergebnisse vergrössern die Möglichkeit des Zurücktretens der Alkaloide in den Stoffwechsel jedoch ebensowenig wie das Studium der Lokalisation.

Der ziemlich hohe Gehalt der Blattalkaloide in den eben abgefallenen Blättern lässt sich mit diesem Zurücktreten ebenfalls schwerlich in Einklang bringen, die Quantität pro 1 cm^2 Blattoberfläche ist zwar etwas kleiner als im erwachsenen Blatte, aber durchaus nicht null, sowie von einem von uns für das Koffein in den Thea- und Coffeablättern nachgewiesen worden ist.

Aus dieser Arbeit muss also geschlossen werden, dass die Blattalkaloide der Cinchona succirubra bei den stets in den Blättern stattfindenden Dissimilationsprozessen entstehen und dass ihre Quantität besonders da zunimmt, wo diese Prozesse die Oberhand gewinnen und Transport nach der Rinde unmöglich ist.

Weil bei den an der Pflanze alternden Blättern die Alkaloide nicht zusondern sogar abnehmen, liegt die Folgerung auf der Hand, dass von einem Alkaloidtransport nach der Rinde die Rede sein muss, womit vielleicht eine Umbildung in das Chinin und seine Nebenalkaloide verbunden sei.

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[1]) C. SPRUIT P.Pzn., Over de toepassing van een quantit. methode ter beoordeeling van de productiviteit van kinaplanten. Jaarverslag Vereeniging Proefstation Personeel 1925.

Botany. — *On the connection between the geotropic curving and elasticity of the cell-wall.* By R. HORREÜS DE HAAS. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of March 31, 1928).

As the principal cause of growth and geotropic curves SACHS and DE VRIES looked upon the pressure put upon the cell-wall by the cell-contents. Though DE VRIES knew that two factors have to be considered viz. the elasticity of the wall and the osmotic value of the cell-fluid, he has laid too much weight upon the production of osmotic active matter.

It was soon proved however that at the convex side a decrease rather than an increase of the quantity of osmotic active matter took place. Moreover the curve of unicellular organs remained unexplained. Consequently the opinion was repeatedly expressed, that the cause of this curving was to be looked for not so much in the change of the quantity of osmotic active matter as in a change of the elasticity of the cell-wall.

As the suctive power of the cells at the convex side increased, although the suctive power of the contents remained constant, a lessening of the wall-pressure had to be thougt of.

From experiments made by OVERBECK (1926) it is proved that this lessening of the wall-pressure cannot be the consequence of active growth of the cell-wall, because the curving proceeds normally also by temperatures by which the active growth is absolutely eliminated. So it must be the consequence of an enhanced elasticity of the cell-wall.

I have succeeded in proving this difference in elasticity by dividing length wise into halves embryo-roots of Vicia Faba, after delivering them from the strained position in which they had been lying for twelve hours horizontally, in which position they were of course prevented from curving.

Immediately strong curving of the upper-half (convex-half) appeared, while the bottom-half (concave-half) did not curve, or even somewhat in the opposite-direction of the convex-half. Since we may believe with URSPRUNG and BLUM (1924) that the osmotic value of the cells of upper- and netherhalf is equal, we must draw the conclusion that there is a lessening of the elasticity passing from the convex- to the concave-side.

This difference in elasticity between both sides of the root could also be proved by direct measure. These experiments in ductility were made with a balance of which one scale had been removed. This scale had been replaced by a wooden catch. A similar catch had also been fixed

to the bottom of the balance. If a divided root was placed between those two catches, then it was possible by burdening the remaining scale to bring about a certain stretching. The extent of this stretching was observed in m.m. movement of the balance needle.

In this way it could be ascertained whether the same load caused unequal lengthening of convex- and concave half.

As the subjoined list proves the upper half of the root possessed a much greater elasticity than the nether half. With the same load (15 gr.) the following movement of the balance needle was observed.

For upperhalf.	netherhalf.
1.5 mm.	1.0 mm.
2.0 ,,	1.5 ,,
0.9 ,,	0.7 ,,
1.0 ,,	0.5 ,,
1.1 ,,	0.9 ,,
0.9 ,,	0.6 ,,

If on the otherhand the root was divided into a left and a right half, then, as indeed might be expected, no difference in elasticity was found.

Movement of the balance needle by equal load:

For left half.	For right half.
0.5 mm.	0.5 mm.
0.75 ,,	0.7 ,,
0.7 ,,	0.7 ,,
0.7 ,,	0.65 ,,
0.7 ,,	0.7 ,,

How is the connection between the geotropic stimulus and the increased elasticity of the cell-walls to be imagined?

An indication for it is to be found in the researches of WENT JR. (1927) who furnishes the proof of the constant production of a "growing matter" in the tops of coleoptiles of Avena. Without the presence of this "growing matter" no growth. WENT JR. now pre-supposes the production of the phototropic curving the consequence of an unequal division of the normal quantity of "growing-matter" over both sides of the coleoptiles.

To prove the connection between increased elasticity of the cell-wall and the quantity of growing-matter, experiments in elasticity were made by myself with coleoptiles of Avena, after removing the first leaf, in the same way as has been described above for Vicia Faba. Some of the Avena plants with which the work was done, were decapitated, and after a lapse of two hours again decapitated. Hereby nearly all the growing-matter in these plants was removed from the coleoptile. As is

evident from the subjoined list a considerably greater elasticity was found in the non-decapitated material i.e. for the coleoptiles with "growing-matter" than in the material void of growing-matter.

By burdening with 15 gr. the following movement of the balance-needle was observed.

In non-decapitated plants.		In decapitated plants.	
0.8 mm.		0.7 mm.	
0.8 ,,		0.6 ,,	
1.0 ,,		0.65 ,,	
0.7 ,,		0.7 ,,	
0.8 ,,		0.7 ,,	
1.0 ,,	average	0.7 ,,	average
1.0 ,,	0.87 mm.	0.65 ,,	0.66 mm.
0.9 ,,		0.7 ,,	
0.9 ,,		0.75 ,,	
0.9 ,,		0.6 ,,	
0.9 ,,		0.7 ,,	
0.8 ,,		0.6 ,,	
		0.65 ,,	
		0.6 ,,	

In analogy with the statement of WENT Jr. given for the phototropic curving we can form for ourselves the following idea of how the geotropic curving is brought about.

The geotropic curving depends upon the unequal division of the quantity of "growing-matter". Consequently a larger quantity reaches the convex than the concave side.

The unequal division of the quantity of growing-matter causes unequal elasticity of the wall on the opposite sides.

The turgorenergy now brings about the curving. Subsequently the curving is fixed by over-stretching or active growth.

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